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Títol: Feasibility study of an optical quantum computing laboratory under the EETAC framework

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Títol: Estudi de viabilitat d'un laboratori òptic de computació quàntica sota la infraestructura de l'EETAC

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Resum

La tecnologia quàntica és un dels camps més prometedors i, a la vegada, desafiadors del segle XX; aprofita algunes de les propietats de la mecànica quàntica, especialment l'entrellaçament quàntic i la superposició, per posar en marxa aplicacions pràctiques i útils com la computació i la criptografia quàntiques, entre d'altres. No obstant, els mitjans per aconseguir-ho fins fa poc eren només assequibles per estudiants avançats de determinats centres superiors d'estudis. Pel que fa a l'EETAC, compte dins del mateix Campus amb l'Institut de Ciències Fotòniques, on no només es dediquen a la investigació i al desenvolupament sinó que també s'ofereix un ampli ventall d'estudis enfocats a ampliar els coneixements en ciències i tecnologies òptiques. D'altra banda, al quadrimestre 4A del Grau en Enginyeria de Sistemes de Telecomunicacions i Telemàtica, s'imparteix l'assignatura Tecnologies d'Informació Quàntica, on es descriuen els principals conceptes quàntics i l'aplicació dels mateixos en els àmbits de la computació quàntica i comunicació quàntica. Tenint en compte els grans avantatges que aquesta tecnologia ofereix, aquesta memòria pretén donar una visió general dels conceptes bàsics de la computació i l'òptica quàntiques mitjançant explicacions del conceptes bàsic amb la finalitat de dur a terme experiments a nivell universitari que apropin la tecnologia de la computació i la informació quàntica als estudiants. El estudi realitzat proporcionarà les condicions, característiques i requisits dels elements òptics necessaris per realitzar aquests experiments així com també de diferents proveïdors dels mateixos. Finalment es proporcionaran el pressupostos que suposaria per la nostra Escola posar en marxa aquesta innovadora iniciativa en l'àmbit de les tecnologies de la informació quàntica.

Título: Estudio de viabilidad de un laboratorio óptico de computación cuántica bajo la infraestructura de la EETAC
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Fecha: 2017

Resumen

La tecnología cuántica es uno de los campos más prometedores y, a la vez, desafiantes del siglo XX; aprovecha algunas de las propiedades de la mecánica cuántica, especialmente el entrelazamiento cuántico y la superposición para poner en marcha aplicaciones prácticas y útiles como la computación y la criptografía cuánticas, entre otras. No obstante, los medios para conseguirlo hasta hace poco sólo estaban al alcance de estudiantes avanzados de determinados centros superiores de estudios. La EETAC cuenta dentro del mismo Campus con el Instituto de Ciencias Fotónicas donde no sólo se dedican a la investigación y al desarrollo sino que también se ofrece un amplio abanico de estudios enfocados a ampliar los conocimientos en ciencias y tecnologías ópticas. Por otro lado, en el cuatrimestre 4A del Grado en Ingeniería de Sistemas de Telecomunicaciones y Telemática se imparte la asignatura Tecnologías de Información Cuántica donde se describen los principales conceptos cuánticos y la aplicación de los mismos en los ámbitos de la computación cuántica y comunicación cuántica. Teniendo en cuenta las grandes ventajas que esta tecnología ofrece, esta memoria pretende dar una visión general de los conceptos básicos de la computación y la óptica cuánticas mediante explicaciones de los conceptos básicos con la finalidad de llevar a cabo experimentos a nivel universitario que acerquen la tecnología de la computación y la información cuántica a los estudiantes. El estudio realizado proporcionará las condiciones, características y requisitos de los elementos ópticos necesarios para realizar estos experimentos así como también diferentes proveedores de los mismos. Finalmente se proporcionará el presupuesto que supondría para nuestra Escuela poner en marcha esta innovadora iniciativa en el ámbito de las tecnologías de la información cuántica.

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Overview

Quantum Technology is one of the most promising and, at the same time, challenging fields of the twentieth century; it takes advantage of some of the quantum mechanics properties, specially quantum entanglement and superposition to start up useful applications as quantum computation and quantum cryptography, among others. However, not long ago, the material means to achieve this end were only available for a few advanced students of higher education academic centers. The EETAC counts with the Institute of Photonics Sciences (ICFO) in the same Campus which does not only focus on investigation and development but also offers a wide variety of studies centered on broaden the knowledge on science and optical technologies. On the other hand, in the 4th-A semester of the Bachelor's Degrees on Telecommunications Systems Engineering and Telematics Engineering the subject Quantum Information Technologies is taught and the main quantum concepts and their applications in quantum computation and quantum communication are given. Taking into account all the outstanding advantages that this technology offers, this study expects to give an overview on the quantum computing and quantum optics basic concepts through simple explanations with the objective of carrying out experiments in the undergraduate level able to bring quantum computation and quantum information technologies closer to the students. The study carried out will provide the conditions, characteristics and requirements of the optical elements needed for the experiments as well as different providers. Finally the required budget for our School to start up this innovative initiative in the quantum information technologies will also be given.

Agradecimientos

Quisiera agradecer, en primer lugar, a los directores de este trabajo, Pere Bruna y Santiago Torres, su esfuerzo y dedicación para su culminación. Ha sido un largo periodo de cooperación a través de correos electrónicos y reuniones en la universidad donde siempre se han mostrado comprensivos y dispuestos a compartir conmigo su conocimiento. Sin su ayuda no habría sido posible.

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LIST OF ACRONYMS

Acronym	Meaning
APD	Avalanche photodiode
BBO	Beta Barium Borate
BB84	Bennett Brassard 1984
CCU	Coincidence Counting Unit
CNOT	Controlled NOT
EETAC	Escola d’Enginyeria de Telecomunicació i Aerospacial de Castelldefels
EPR	Einstein, Podolsky and Rosen
FPGA	Field Programmable Gate Array
HWP	Half wave plate
ICFO	Instituto de Ciencias Fotónicas
IEC	International Electrotechnical Comission
IR	Infrared
NA	Not Applicable
PBS	Polarizing Beam Splitter
QWP	Quarter wave plate
SPCM	Single photon counting module
SPDC	Spontaneous parametric down-conversion
TIQ	Tecnologies d’Iniformació Quàntica
UV	Ultraviolet

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CHAPTER 1. INTRODUCTION

1.1. Motivation

Quantum technology is one of the most promising and challenging fields in nowadays science. Particularly, quantum computing, quantum cryptography or generally speaking, quantum information technologies claim to change in short term our paradigm of classical computing and communications. So, it is not surprising, to find these topics included in the current syllabus of many undergraduate or postgraduate studies more and more frequently. However, the requirements to perform even the simplest experimental test in these areas are far beyond the budget and capabilities of typical public institutions. Nevertheless, technology has moved forward to the point where definitely modern experiments that explore the fundamentals of quantum mechanics are approachable to undergraduates. The equipment and techniques have become cheaper and more manageable so that they are feasible to be introduced into the undergraduate laboratory. Assembling an undergraduate quantum setup could seem an arduous task but being clear about the experiments to perform it is just a matter of time and a bit of skills to gather all the required components and start doing measurements. It is, of course, important to count on the help of experienced professionals in order to ensure that the setup will work as expected and let us retrieve reliable data.

Under this framework, the general goal of the present thesis was to obtain a structured as well as simple and useful guide to perform basic experiments in the undergraduate level that could be set up on an optical bench located in an adapted room for this purpose. The initial idea was implementing an experiment that gave as a result entangled photons because of the size of the setup required and the variety of experiments that can be performed just by switching the same equipment. Quantum mechanics are such an abstract subject that may be difficult to understand at once, so these experiments are intended to put into practice the related knowledge acquired during lectures while creating an active learning environment, fundamental to effective education.

This is not the first time such an initiative is driven. Enrique J. Galvez, professor of physics and astronomy at Colgate University in Hamilton, New York, has focused his studies on optics and quantum mechanics offering an interesting and motivating portfolio of activities for undergraduates which have also as the main purpose teaching in a practical and understanding way the principles of quantum mechanics. Needless to say he is not the only one on promoting this good practice. Many other universities around the world have also joined to this proposal giving interesting as well as reliable results. Some examples are Whitman College in Washington or the University of Vienna which combines a mix of research areas from fundamental quantum physics to new quantum technologies.

1.2. Quantum computing: state-of-art

Instead of using bits, quantum computing operates with quantum bits called qubits. Qubits are different from regular bits because they do not just have two defined states but plenty others. This ability to exist in multiple states opens a big door of possibilities for the way computers operate nowadays. The calculation process ends by making a measurement which collapses the system in one of the possible states. The basic and distinguishing feature of a quantum computer is its ability to operate simultaneously on a collection of states, thus potentially performing many operations in the time a classical computer would do just one.

Quantum algorithms are most of the times probabilistic so the given output is the one with the highest probability. They take advantage of the ability of subatomic particles to exist in more than one state at any time. All this leads us to figure out the huge impact this technology has for improving the way many fields work. One of these fields is without a doubt communications. Nowadays we live in such a connected world that makes it indispensable to communicate in a secure way and the way to ensure safe communications is encryption as it provides a fundamental shield that protects confidential data. The most secure and widely used methods to protect the integrity and confidentiality of data transmission are based on symmetric cryptography whose security relies on a basic premise: factoring a large prime number.

Factoring an integer is the basis of the most common encryption schemes. RSA – named after Rivest, Shamir and Adleman (see [1]) – is one of the first public key cryptosystems widely used for secure data transmission. It works in such a way that the encryption key is public and different from the decryption one which is kept secret. RSA scheme assumes that it is easy to obtain the product of two large prime numbers but it is highly much more difficult going back and factoring the product for obtaining the original primers. From RSA point of view factoring large prime numbers is computationally unmanageable so its basic principle is based on the practical difficulty of factoring the product of two large prime numbers. This assumption is only valid for classical computers since Shor's algorithm shows that factoring could be possible on an ideal quantum computer that would surely threaten the current security cryptosystems. Shor's discovery assumes that quantum computers are able to achieve this feat thanks to having hardware capable of processing every possible answer simultaneously in polynomial time instead of investing a large exponential amount of time as with classical computers (see [2]).

To the light of the aforesaid, the step forward would be building a quantum computer whose construction rendered public key cryptosystems insecure. In terms of the components industry there is a prediction settled by an American businessman called Gordon Earle Moore who formulated Moore's law in the 70s (see [3]). Moore stated that the number of transistors in an integrated circuit – which means processor speed – would double every year for the next 10 years. This prediction has been achieved for several decades and has also been a guide for setting targets in the semiconductor industry which have been possible to achieve due to the unusual quality of transistors of getting better as they get smaller. A small transistor can go from status on to off with less power and at greater speed when compared to bigger ones but quantum effects are beginning to interfere in the functioning of electronic devices as they are made smaller (see [4]).

A quantum computer operates by setting the qubits in a perfect trend that faces any existent problem by managing qubits in a logic way called quantum algorithm. Qubits can store much more information than classical bits as they operate according to two key principles of quantum physics: superposition and entanglement. Superposition means that each qubit can represent both a 0 and a 1 at the same time while entanglement means that qubits can be correlated with each other, that is, the state of one qubit – whether it is a 0 or a 1 – can depend on the state of another. Using these two principles, qubits can act as sophisticated switches that enable quantum computers to function in ways that allow them to solve difficult problems intractable for today's computers (see [5]). In fact, one of the most exciting things about quantum computation and quantum information is how little is known about what makes quantum computer more powerful than classical computers, the class of problems they can solve or how that kind of problems can compare to the ones that can be solved efficiently on a classical computer. In addition, coming up with good quantum algorithms seems to be hard because designers face two difficult problems not faced for classical computers: The first one is designing quantum algorithms thinking of truly quantum effects to achieve the desired algorithm end as human intuition is set to the classical world. The second one is that the new quantum algorithm must be better than any existing classical algorithm. It is possible that one may find an algorithm that uses quantum aspects but that is not of interest because there is an already implemented classical algorithm with comparable performance (see [6]).

With the objective of going forward with quantum technology and trying to give answers to all this questions the first quantum computing company was born, D-Wave Systems (see [7]). They have been the creators of D-Wave 2000Q known as the most advanced quantum computer in the world and also proved to be 100 million times faster than a conventional computer by a team of scientists of Google and Nasa (see [8]). Its principal goal is bringing practical quantum computing to reality. Among the multiple applications it can manage one can find the optimization on water network, machine learning, object detection, financial analysis, etc. This means we have at hand such a powerful computer capable of giving good answers to several daily problems in a short period of time. But moving quantum computing to an industrial scale is difficult: One problem with quantum computers is that when trying to look at the subatomic particles (atoms, ions, photons or electrons) they can collapse into one of all their possible states. It is not necessary to look to all the particles in the system, just by looking to one qubit it will assume the value of 0 or 1 and that will cause the other qubits to also bump because of the two properties abovementioned. Then, in order to develop a practical quantum computer, scientists have to develop ways of making indirect measurements for preserving the system's integrity. In addition, it is also important to note that there are many experts that differ from the opinion that the D-Wave is actually the first quantum computer and, therefore, that it is really capable of implementing the quantum algorithms known so far.

For now, the technology required to develop quantum computers is still beyond our reach. Most research in quantum computing is very theoretical. The most advanced quantum computers have not gone beyond manipulating more than 16 qubits meaning that they are far from practical applications. Computations occur when qubits interact with each other, therefore for a computer to work properly and efficiently it needs to have many qubits. Nevertheless, IBM has recently announced – March 2017 – its Q division is developing quantum computers that could be sold commercially within the upcoming years. They claim commercial quantum computer systems with approximately 50 qubits will be produced in the next years and assure they will exceed the speed and computing capacity of the most powerful current computers (see [9]).

It is also worth mentioning that last year the same company – IBM – created a quantum processor and also set up a platform so that any expert or researcher worldwide could test it. This processor is composed by five qubits – which means a record for the company – and whose purpose is enabling new scientific discoveries and innovations (see [10]). Moreover, there are other noticeable experiments that show the growing importance of quantum technologies worldwide and for the future: An international team of researchers demonstrated quantum teleportation between two Canary Islands. Teleportation relies on entanglement and leads to secure communications. The researchers managed the successfully teleporting of photons over a distance of 143 km and published a paper in Nature (see [11]).

According to the paper, it was not an easy task as it took nearly a year to develop systems to handle some of the weather conditions because using optical fibers caused great losses of the transmitted signal. The chosen tool was a laser that helped teleport a photon from one Canary Island to the other. Teleportation is not taking a photon and making it disappear and reappear in another place as popular sci-fi culture states. Quantum teleportation consists of transmitting the information contained in a photon’s quantum state to another thanks to the phenomenon of quantum entanglement without actually travelling any distance. The transfer of information takes place when the sender measures the quantum state of their photon making the receiver’s entangled photon instantly collapse. However, for understanding the information, the receiver has to know what the original measurement was together with some other instructions that are sent via normal communications.

Quantum teleportation is an extremely delicate process that may take several years to produce any kind of practical communication device but it shows how useful quantum technologies can be for practical applications in the future. Recently, a group of scientists launched the world’s first quantum satellite from China with the objective of developing a communications system that cannot be cracked by hackers. One of the tasks during the satellite’s mission will be to send coded communications back to Earth that cannot be read by eavesdroppers. The intention is transmitting information using photons because it cannot be analyzed by third-parties without the right codes.

The idea is taking advantage of the capability of photons of being able to carry information to perform and experiment for which China will send encrypted data to Vienna using normal communications and the password to unlock this data will be sent using the satellite – the quantum system. This means that if a third party tried to find out the code their intention will be discovered and the code destroyed. If the system succeeds it could open up a means to communicate over long distances.

The importance of quantum technologies is evident; the current investigations lead to think this way unavoidably. The number of publications in this regard has been high within the last 20 years according to Cornell University. In their webpage there is a useful tool for searching among all the papers that have been submitted and classified by subject area. Doing a quick search the number of results obtained for this year 2017 and for the subject Quantum computing is 50. The available papers belong to different areas such as Cryptography and Security (Limitation on Transversal Computation through Quantum Homomorphic Encryption), Logic in Computer Science (Quantum Field Theory and Coalgebraic Logic in Theoretical Computer Science) or Emerging Technologies (Geometry-Based Optimization of One-Way Quantum Computation Measurement Patterns). The average of publications from 2007 to 2016 is 114 per year which is quite interesting from the technologic point of view as this means there is a lot of research on the field and the day quantum computers are a reality is near.

Quantum computation and quantum information has let us discover they way to think physically about computation and that this approach hides many new and exciting capabilities for information processing and communication. These two new paradigms offer challenges to explore to physicists and scientists worldwide and the fruits of these explorations may one day result in information processing devices with capabilities far beyond today's computing and communication systems with collateral benefits for the society we live in (see [12]).

Finally, it is worth mentioning that here, in the Campus, there is an institution whose aim is making progress in different fields of science and technology using light as a basis, namely laser light because it is one of the most enabling technologies currently available. This institution is ICFO (Instituto de Ciencias Fotónicas). The current researchers of ICFO are talented PhD students and post-doctoral researchers who aim to achieve important goals by working in different projects focused on problems in Health, Energy, Information, Safety, Security and the Environment. They also organize different courses and seminars for training scientists and technicians. One of the ongoing projects at ICFO is related to Quantum Information Theory, they study quantum systems related to entanglement considering a variety of quantum states and both regular and complex networks. They also offer opportunities for personal and professional growth through different programs as master studies in optics and photonic related areas that provide the knowledge to educate future specialists as well as promote the entrepreneurial activity amongst students. There is also a wide variety of courses that give a fundamental background and an overview of the latest advances in the corresponding topics such as Nanophotonics, Quantum Optics, and Quantum Information Technologies.

In order to encourage the next generation of researchers or even to bring the undergraduates closer to this leading topic, ICFO is an important resource within reach to take advantage for implementing different experiments to see all the possibilities that quantum technologies may offer. Currently, one of the Bachelor 's degrees available is the degree in Telecommunication Systems that allows the students to learn the fundamentals and applications for designing, implementing and operating radio systems and even optical communications. The curriculum of the fourth school year of the degree includes a subject called Quantum Information Technologies (TIQ) whose content starts with an introduction to Quantum Physics and continues with Quantum Computing and Quantum Communications. Having into consideration the undeniable relevance of quantum technologies nowadays and for the future it would be really interesting to count on the expertise of the professionals at ICFO and the ones at EETAC for organizing some activities in the undergraduate level with the objective of bringing into practice the basic quantum concepts taught during lectures for the students to be aware of their possibilities and for waking up their interest in the field.

1.3. Goals and structure of the thesis

The subject TIQ in the semester 4A of the bachelor degree in Telecommunication Systems Engineering describes the main quantum concepts, gives some important definitions and shows to the students the way to operate with qubits and how to handle them by means of quantum circuits using quantum logic gates. For bringing to reality all these concepts that may seem abstract for a beginner the implementation of the proposed quantum optical laboratory will be really useful. Moreover, as quantum technology aims to become useful for many applications as cryptography or quantum computing this laboratory will be an important source to take advantage of.

As previously stated, our general goal is to provide a feasibility study and prepare a structured as well as simple and useful guide to perform basic experiments in the undergraduate level that could be set up on an optical bench located in an adapted room for this purpose under the EETAC framework. The EETAC owns suitable spaces for building a quantum optical laboratory addressed to undergraduate students. This possibility could be taken into consideration after a detailed study of a budget as this is a great chance for promoting quantum technologies that represent the future of science.

This thesis also provides such a budget as well as a relevant explanation on qubits, its notation and the definition of important concepts as polarization, superposition and entanglement. After the corresponding description of the basic concepts, the setup for carrying out our own experiment is also defined along with all the necessary optical components and providers for its assembling.

CHAPTER 2. QUANTUM COMPUTING

2.1. Qubit's basics

This chapter is intended to review the basis of quantum algorithms starting from the basic quantum units of information – qubits – and quantum circuits which are responsible of qubits manipulations by means of different quantum gates (see [12] [13])

The qubit is the basic unit of information analogue to bits but at a quantum stage. Just as bits, qubits have two possible states - $|0\rangle$ and $|1\rangle$ - but the essential feature that makes them infinitely different from bits is that also any linear combination of these two states is actually possible. This unique characteristic is called superposition. In general, the state of a single qubit is represented as:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad \textbf{(2.1)}$$

where α and β are complex numbers, also called qubit amplitudes. Quantum states are denoted by means of ket notation or Dirac notation, where "$|\ \rangle$" is the standard notation for depicting states in quantum mechanics. The states $|0\rangle$ and $|1\rangle$ are orthonormal and together form a basis, commonly called computational basis. The fact of measuring a qubit modifies its current superposition of states making it collapse from the superposition to one of its possibilities, either $|0\rangle$ or $|1\rangle$. Qubits store information that can be altered by using quantum gates.

When trying to retrieve the value out of a quantum state, the result is either 0, with probability $|\alpha|^2$, or 1, with probability $|\beta|^2$. Since probabilities must sum to one these amplitudes meet the normalization criteria represented in the following expression:

$$|\alpha|^2 + |\beta|^2 = 1 \quad \textbf{(2.2)}$$

Suppose two qubits at disposal. In this case, the representation of the corresponding superposition of states will be the one in **(2.3)** which means that four possible combinations exist: $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$**.**

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle \quad \textbf{(2.3)}$$

Comparable to the case of a single qubit, the measurement of this new quantum superposition will turn out | α_{ij} $|^2$ which is the probability of measuring any of the four combinations. An important aspect to consider is that measuring the probability of obtaining one of the possible states of only one of the qubits is feasible. For example, think of obtaining the probability of measuring the first qubit to be in state 0. The two states that satisfy this condition are $|00\rangle$ and $|01\rangle$ because the first qubit of both is 0 so they both must be taken into consideration. As α_{00} and α_{01} are the amplitudes of the possible states that contain 0 for the first qubit the result we are looking for is $|\alpha_{00}|^2 + |\alpha_{01}|^2$ and the post-measurement state will be

$$| \psi\rangle = \frac{\alpha_{00} \; | \; 00\rangle \; + \; \alpha_{01} \; | \; 01\rangle}{\sqrt{|\alpha_{00}|^2 + |\alpha_{01}|^2}} \tag{2.4}$$

The post-measurement state should be normalized, that is, the corresponding amplitudes must be equal to 1. For meeting this condition the post-measurement state should be divided by the factor $\sqrt{|\alpha_{00}|^2 + |\alpha_{01}|^2}$ in order to satisfy the normalization condition the same way the original state does.

The quantum superposition of states for n qubits can be written as follows:

$$|\psi\rangle = \alpha_{00\ldots0}|0_1 0_2 \ldots 0_n\rangle + \alpha_{00\ldots1}|0_1 0_2 \ldots 1_n\rangle + .. + \alpha_{11\ldots1}|1_1 1_2 \ldots 1_n\rangle \tag{2.5}$$

The number of states increases potentially as 2^n. We can see the huge potential of qubits, given that for n = 500, the number of possible superposition states is bigger than the number of atoms of the whole universe.

Returning to 1-qubit, one other practical and useful representation is by means of the Bloch sphere. Since the amplitudes are normalized to one a sphere of radius 1 can be defined. Any quantum state can be written by means of sine and cosine (see **(2.6)**) where the phases θ and Φ determine a certain point in the sphere where the state is placed (see [14]).

$$| \psi\rangle = \cos(\theta/2) \; | \; 0\rangle + e^{i\Phi} \sin(\theta/2) \; | \; 1\rangle \tag{2.6}$$

Then, for representing the state $|0\rangle$ θ must be equal to 0 while for obtaining the state $|1\rangle$, θ must be π rad (see Figure 2.1).

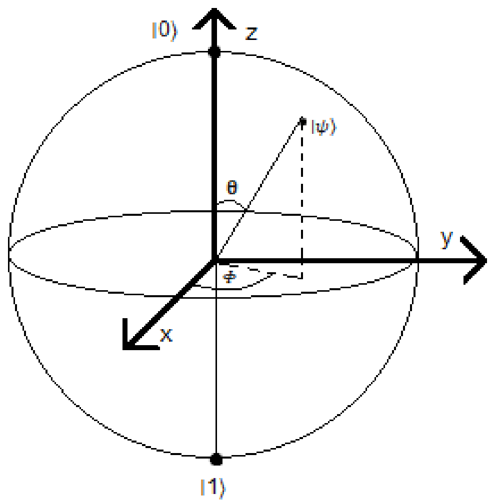

Figure 2.1. Bloch sphere

Qubits have many other properties that make them unbeatable when compared to bits. One of these properties is the entanglement. Quantum entanglement is the relationship between separated quantum systems. In other words, two particles can be intimately linked to each other no matter how apart they are as soon as they are entangled. A change to one of them will immediately affect the other. Bell states or EPR pairs are the simplest examples of this phenomenon. EPR is an acronym for the names Einstein, Podolsky and Rosen (see [17]). As their name indicates each Bell state is composed by two qubits in the way $|q_1q_2\rangle$.

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \quad \textbf{(2.7)}$$

$$|\beta_{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \quad \textbf{(2.8)}$$

$$|\beta_{10}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \quad \textbf{(2.9)}$$

$$|\beta_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \quad \textbf{(2.10)}$$

Given a certain EPR pair (see **(2.7), (2.8), (2.9)** and **(2.10)**) one can do a measurement on one of the qubits making the other qubit collapse immediately.

For every pair the probability of obtaining any of the two possible states is 50%. For instance, in the case of the state $|\beta_{00}\rangle$ if the result of measuring the first qubit is $|0\rangle$ then, the second qubit will immediately also collapse to $|0\rangle$. No matter how apart in the universe both qubits are, the fact of measuring one of them will affect the other as they are entangled.

2.2. Quantum gates

In order to change the properties of a qubit quantum gates are needed. Quantum gates are responsible of managing all the transformations applied to any superposition of quantum states. These gates are analog to conventional logic gates in use for conventional computers but they are infinitely more powerful because they operate qubits which store far more information than classical bits.

Quantum gates have one important and remarkable peculiarity that makes them unrivalled when compared to classical logic gates: they are reversible. This unequaled feat is due to the fact that quantum logic gates can be represented by unitary matrices – in order to preserve the normalization – and the inverse of a unitary matrix is a unitary matrix too. So, consequently, a quantum gate can always be reverted by another quantum gate. Quantum gates can operate over n qubits. The most common quantum gates can operate over one or two qubits and can be represented by two by two or four by four matrices respectively as explained in the next subsections. The dimension of the matrix can be written as 2^n x 2^n.

2.2.1. Single qubit gates

Imagine a process capable of taking $|0\rangle$ and replacing it by $|1\rangle$ and vice versa. The single qubit gate able to do such transformations is the quantum NOT gate (see [13]) whose representation is the following:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.11}$$

The quantum NOT gate is one of the most used single qubit gates and together with three others forms the so called Pauli matrices. They are a set of four two by two matrices named after the physicist Wolfgang Pauli.

In order to get the effect of a NOT gate acting over a generic qubit we can use vector notation. The result will be as shown below:

$$X \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix} \tag{2.12}$$

Then, the corresponding vector notation to our example state is $\begin{pmatrix}\alpha \\ \beta\end{pmatrix}$ which means that, in vector notation, $|0\rangle$ will be represented as $\begin{pmatrix}1 \\ 0\end{pmatrix}$ and, analogously, $|1\rangle$ will be $\begin{pmatrix}0 \\ 1\end{pmatrix}$. As seen in **(2.12)** the effect of an X-NOT gate acting on a qubit implies a swapping of the amplitudes.

Unlike the X gate that only executes a single change the Y gate represented in **(2.13)** performs two actions on the qubits: firstly, it adds a phase of 90 degrees and also shifts the amplitudes of both states.

$$Y = \begin{pmatrix}0 & -i \\ i & 0\end{pmatrix} \qquad \textbf{(2.13)}$$

The next Pauli matrix is Z – **(2.14). It** just adds a phase of 180 degrees to the state $|1\rangle$ while $|0\rangle$ remains unchanged.

$$Z = \begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix} \qquad \textbf{(2.14)}$$

Finally, the last of Pauli matrices is I, also known as repeater, represented in **(2.15)**. It does not perform any change to the original state. It is a two by two identity matrix.

$$I = \begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix} \qquad \textbf{(2.15)}$$

The table below aims to gather Pauli matrices together with their effect on a generic qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$.

Table 2.1. Effect of Pauli matrices over a generic qubit

Matrix	Action	Result
$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$	$\begin{pmatrix} \beta \\ \alpha \end{pmatrix}$
$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$	$i\begin{pmatrix} -\beta \\ \alpha \end{pmatrix} = e^{i\pi}\begin{pmatrix} -\beta \\ \alpha \end{pmatrix}$
$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$	$\begin{pmatrix} \alpha \\ -\beta \end{pmatrix}$
$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$	$\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

After having introduced Pauli matrices the next single qubit worth describing is the Hadamard matrix.

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \equiv \frac{1}{\sqrt{2}}(X+Z) \quad \textbf{(2.12)}$$

For explaining its behavior let $|\mathrm{q_{in}}\rangle = \alpha\,|\,0\rangle + \beta\,|\,1\rangle$ be the input state. Then the result of the action that the Hadamard matrix executes over it can be written as

$$|q_{out}\rangle = H|q_{in}\rangle$$

(2.13)

$$|q_{out}\rangle = \alpha\frac{|\,0\rangle + |\,1\rangle}{\sqrt{2}} + \beta\frac{|\,0\rangle - |\,1\rangle}{\sqrt{2}} = \alpha|+\rangle + \beta|-\rangle$$

The Hadamard gate transforms any input qubit $/q_{in}\rangle = \alpha|0\rangle + \beta|1\rangle$ into the superposition $|q_{out}\rangle = \alpha|+\rangle + \beta|-\rangle$ where |+⟩ and |-⟩

represent a new basis, called Hadamard basis. This means that a generic qubit $|q\rangle$ can also be represented in terms of $|+\rangle$ and $|-\rangle$.

$$|q\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha\frac{|+\rangle + |-\rangle}{\sqrt{2}} + \beta\frac{|+\rangle - |-\rangle}{\sqrt{2}} = \frac{\alpha + \beta}{\sqrt{2}}\Big|+\rangle + \frac{\alpha - \beta}{\sqrt{2}}\Big|-\rangle \tag{2.14}$$

Hadamard matrix represents a change of basis between the computational and the $|+\rangle/|-\rangle$ basis.

2.2.2. Multiple qubit gates

Firstly we will show a quantum state operating over two qubits simultaneously. The most common one is the CNOT-gate (Controlled NOT gate).

This gate has two qubits as input. For illustrating this statement see **Figure 2.2**.

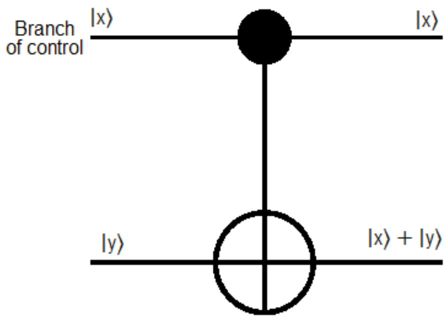

Figure 2.2 Representation of a CNOT quantum gate.

The qubit in the first branch is known as the control qubit and the other is the target qubit.

CNOT will act on the target qubit if and only if the qubit on the branch of control is $|1\rangle$**.** The action it will perform when this condition is fulfilled is reversing the target qubit. The way this matrix will behave depends on the inputs and the outputs according to the notation $|q_{in1}q_{in2}\rangle \rightarrow |q_{out1}q_{out2}\rangle$ will be as follows:

$|00\rangle \rightarrow |00\rangle$**:** The qubit in the branch of control is 0, then, CNOT does not act on the second qubit.
$|01\rangle \rightarrow |01\rangle$**:** The qubit in the branch of control is 0, then, CNOT does not act on the second qubit.
$|10\rangle \rightarrow |11\rangle$**:** The qubit in the branch of control is 1, then, CNOT acts on the second qubit flipping it from 0 to 1.

$|11\rangle \rightarrow |10\rangle$**:** The qubit in the branch of control is 1, then, CNOT acts on the second qubit flipping it from 1 to 0.

It is worth mentioning that the CNOT gate is a controlled gate performing X gate which is one of the Pauli matrices. Analogously one can realize that it is also possible to have other controlled gates such as the controlled Z or the controlled Y gates.

In the same way as classical computing where the NAND gate exists – it is the result of operating an AND gate and a NOT gate, negative AND – in quantum computing any quantum logic gate with more than one qubit at the input may be generated from a quantum CNOT gate and the Pauli matrices.

2.3. Quantum circuits and their applications

Once the most common quantum gates and their performance have been described the next step to take is how they interact together constituting quantum circuits. They are similar to classical computer circuits because their function is manipulating information by means of gates but quantum instead of classical.

The functioning of quantum circuits and qubits could seem similar to classical bits so far – except for superposition. In fact, as qubits are the quantum counterparts for bits the way to understand them much easily is making an analogy between them. However, qubits have many other properties that make them unbeatable when compared to bits. One of these properties is the entanglement. Taking advantage of the multiple peculiarities entanglement offers, several technological applications have emerged in fields such as quantum cryptography or quantum computing as for example teleportation experiments which consist of transferring a qubit between two people – Alice and Bob, sender and receiver respectively.

Bell states can be generated by means of quantum gates using a Hadamard gate and a CNOT gate as shown next**:**

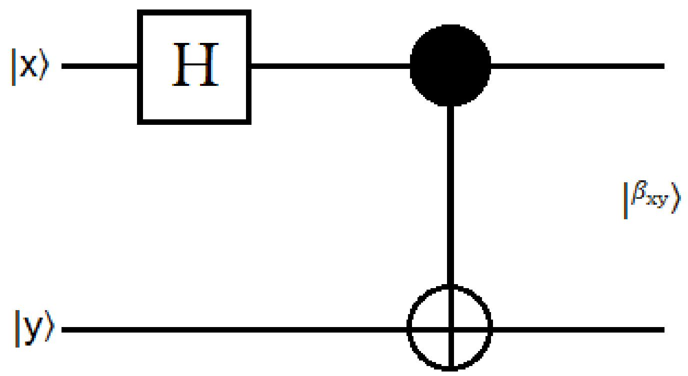

Figure 2.3. Bell states generator

Once the EPR Pairs described a brief explanation on how quantum teleportation is carried out will follow. Consider Alice (sender) and Bob (receiver) who are separated from each other but share an entangled state $|\beta_{00}\rangle$ formed by two qubits as seen in **(2.7).** Alice keeps the first entangled qubit for herself and Bob the second one. As the sender, Alice wants to transfer the state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ which is a qubit in a superposition of states. Now, she own two qubits: one of the EPR pair $|\beta_{00}\rangle$ the other one is the one she wants to send. In order to start the communication she makes use of the quantum gate CNOT using these two qubits as inputs and then applies the quantum gate Hadamard only to $|\psi\rangle$. Once done she measures the outputs and obtains two classical bits – b_1 and b_2 – which are sent to Bob over a classical communication channel (telephone). On his side, Bob uses the transformation $Z^{b1}X^{b2}$on his qubit – Z and X are both Pauli matrices. The result Bob gets will be $|\psi\rangle$. The figure below is a diagram of quantum teleportation just described.

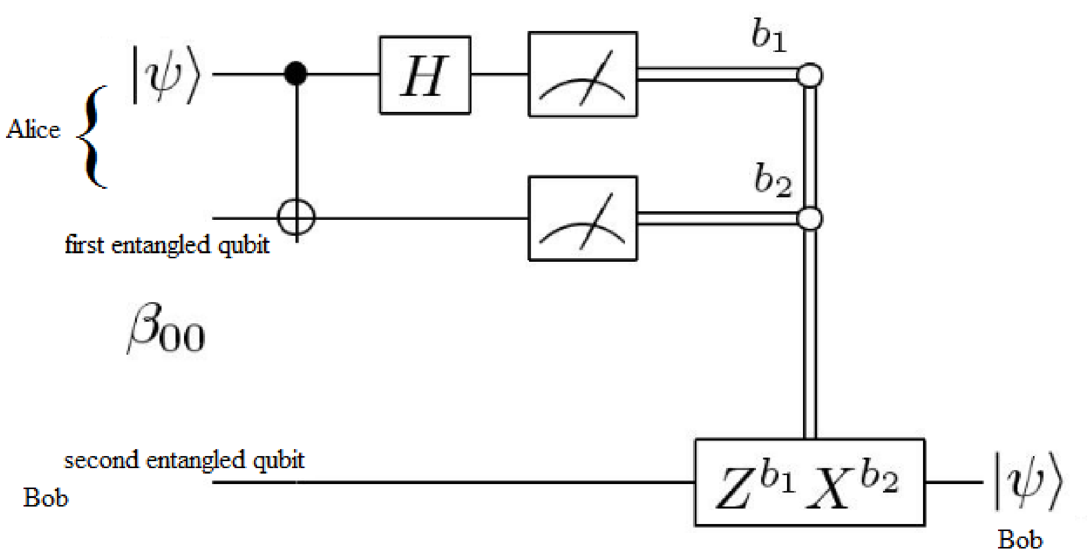

Figure 2.4 Quantum teleportation diagram where $|\psi\rangle$ is the qubit to teleport and $|\beta_{00}\rangle$ is the auxiliary entangled state.

The amazing part about teleportation is that Alice and Bob could use their shared entangled state as a communication channel to make the state $|\psi\rangle$ on Alice's side be communicated to Bob's side wherever in the world he was, Alice does not even have to know his location. The result of Alice's operations over her two qubits – which has four possible outputs with equal probability of ¼ - can be determined by just two classical bits. This principle brings one to think that using quantum states to store information instead of the classical is significantly more advantageous.

Communications are susceptible to be interfered by a third party whether they are classical or quantum. Eavesdropping is to listen secretly to a private conversation so an eavesdropper – Eve – is someone who meddles in the communication between Alice and Bob with the objective of getting information out of it. In the classical world, the exchange of data between Alice and Bob could be encrypted using a secret key but carrying out the communication this way would be safe only once because if they use the same key for encrypting the information several times Eve could manage to decrypt it by analyzing the structure of the messages they are sharing. Even if Alice and Bob generated a new secret key every time they wanted to establish a new communication there is no way to ensure data exchange using classical methods. Quantum entanglement, however, is an outstanding method to beat eavesdropping because Alice and Bob use entangled states and could detect if there is any attempt of eavesdropping.

In general, quantum computing is a powerful tool to explode and offers several advantages to the world we live and the way we live but, mainly, to the way we communicate. The field dedicated to quantum communications has experienced an enormous growth since year 2000: new standards that consider quantum algorithms as a basis have appeared and also new physical implementations for creating a quantum computer capable of employing quantum optics for multiple beneficial purposes for the human being.

CHAPTER 3. QUANTUM OPTICS

Quantum optics is a field that focuses on the treatment of light as a stream of photons instead of electromagnetic waves, the way photons interact and how they can be physically manipulated through optical elements. It is not a new subject since it dates back to 1900-1925 as quantum theory but has really gained relevance during the last part of the twentieth century.

The intention of this chapter is giving a basis of the most common quantum optics elements that will also be used later for carrying out our experiments together with basic concepts that will appear later. The basis of our experiments is the light generated by a source, a laser.

Polarization is the direction of the electric field for a beam of light. A beam of light is composed by a stream of photons that are polarized all together in a certain direction. There are many different types of polarization being the most typical ones the linear and the circular. In linear polarization the vector that describes the electric field keeps a constant direction while for the circular polarization this vector has a movement of rotation at the same time that the wave travels through space. There are two types of circular polarization according to the direction of rotation of the electric field vector: left and right. In the case of the right circular polarization the electric field vector rotates clockwise and for left circular polarization it rotates counterclockwise. Finally when the light is randomly polarized it is called unpolarized (see [22]).

Polarization will help us represent quantum states. If we take the expression in **(2.1)** and set α = 1 and β = 0 we obtain the following:

$$|\psi\rangle = 1|0\rangle + 0|1\rangle = |0\rangle \tag{3.1}$$

For its representation a linear horizontal polarization can be used (see **Figure 3.1**):

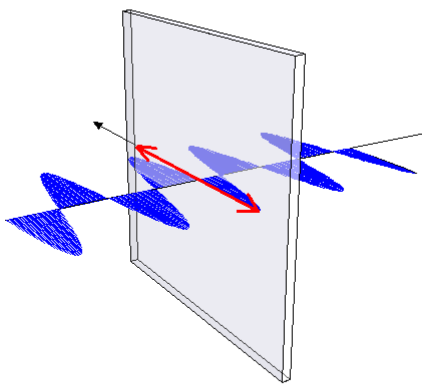

Figure 3.1. Linear horizontal polarization ([23]).

The same procedure will be followed for representing the state $|1\rangle$ but in this case the amplitudes of α and β will be the opposite:

$$|\psi\rangle = 0|0\rangle + 1|1\rangle = |1\rangle \tag{3.2}$$

And a vertical linear polarization will be used to represent it (see **Figure 3.2**).

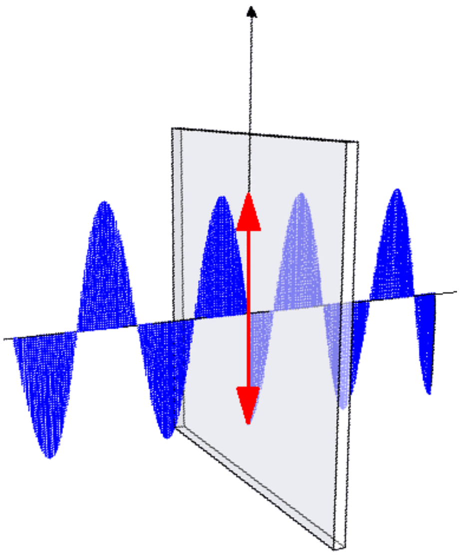

Figure 3.2. Linear vertical polarization (see [23]).

For manipulating qubits represented by different polarization states of the electromagnetic wave different optical elements are needed to create the light and interact with it. We will describe these elements in the following sections.

3.1 Laser

The source of light that will generate the photons for the experiments will be a laser. A laser (Light Amplification by Stimulated Emission of Radiation) will be used to create the electromagnetic radiation needed to generate qubits. Laser operation was first demonstrated in 1960, and since then, they have become essential tools for carrying out optical implementations.

Basically, a laser consists of an optical cavity composed by a gain medium constrained between two mirrors called the output coupler and the high reflector. The laser beam emerges from the output coupler (see [27]).

Depending on the danger that lasers represent there are 7 categories in which they are classified according to the regulation IEC 60825-1 (see [28]). In our case, the lasers that will be used here belong to Class 2 in which visible lasers – from 400 to 700 nm – are included. They are safe elements but good practices and the usage of safety equipment such as protective eyewear is always recommended as metallic tools in the surroundings of the operating laser could redirect the beam emitted by accident.

An alternative way of classifying lasers is according to the nature of its active medium, so, we can find semiconductor, gas (HeNe, Helium-Neon), solid (crystals, altered fibers) or even coloring (Rhodamine 6G) lasers. The most common emitting laser is the diode laser, a semiconductor one, a p-n junction. This kind of lasers presents a wide range of possible applications starting from fiber optic communications and lightning sources to barcode readers or scanning.

Diode lasers produce coherent radiation which means that all the generated photons will oscillate at the same frequency and phase. They present different and useful properties when compared to conventional lasers: small size and weight which turns them into portable electronic equipment, low current, voltage and low power consumption as their performance is not affected by using small battery power supplies.

3.2 Beta Barium Borate

Beta Barium Borate abbreviated as BBO is a common nonlinear birefringent optical material. It is called nonlinear because the polarization P responds in a nonlinear way to the electric field E usually according to a parabolic shape, x^2. This special property has many applications; one of them is parametric frequency conversion that will let us obtain entangled photons in the experiment of entanglement.

Spontaneous parametric down-conversion (SPDC) is an optical effect where the polarization depends quadratically on the electric field and photons incident on the non-linear birefringent crystal are converted into two photons at the output. It is one of the most significant processes in quantum optics as it is mainly used for the production of entangled photon pairs. In SPDC, an incident photon is often known as *pump* while the two outgoing photons are called *signal* and *idler*. This phenomenon is *spontaneous* because the idler and signal photons are naturally produced; there is no specific input beam for generating them. They are created by the combination of the pump beam and the properties of the crystal itself. The adjective *parametric* is given to the process because it depends on the electric fields that interact inside the crystal creating a phase difference between the input and the output. Finally, the term *down-conversion* means the signal and the idler down-converted photons at the output have a lower frequency than the pump. This is a low efficiency process (on the order of 10^{-11}): only a few incident photons are down-converted. The others pass through the crystal unaltered.

The splitting of the pump photons into pairs occurs according to energy conservation laws (see Figure **3.3**).

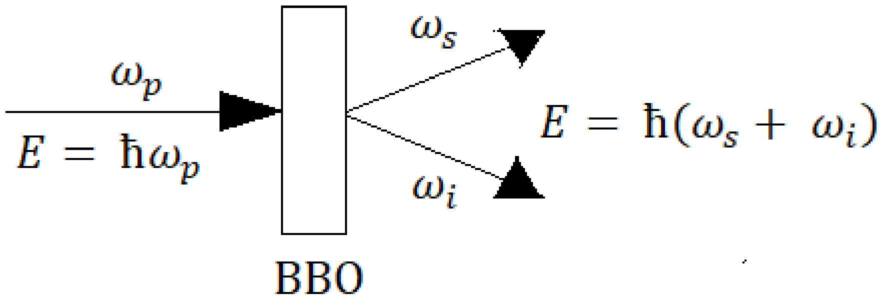

Figure 3.3. Conservation of energy in down-converted photons

Where ħ = h/2**π** (h is Planck's constant) and $\boldsymbol{\omega}_p$, $\boldsymbol{\omega}_s$ and $\boldsymbol{\omega}_i$ are the wavelengths corresponding to the pump, signal and idler (see figure 3.11).

Energy cannot be created nor destructed but it can be transformed. The expressions **3.3** and **3.4** describe this phenomenon.

$$E_i = E_f \qquad \textbf{(3.3)}$$

$$E_k^i + E_p^i = E_k^f + E_p^f \qquad \textbf{(3.4)}$$

In **3.3** the subindices i and f stand for initial and final while in **3.4** k and p stand for kinetic and potential.

In addition to energy conservation, parametric down-conversion also requires the conservation of the photon momentum p = ħk inside the crystal. The momentum conservation can be written as $\overrightarrow{\mathrm{k_p}} = \overrightarrow{\mathrm{k_s}} + \overrightarrow{\mathrm{k_i}}$ which is known as the phase matching condition.

A simple way to explain the functioning of a BBO crystal is by analyzing the way it behaves for a linear input: a BBO is able to convert input linear polarization into output linear polarization but orthogonal with respect to the input (**Figure 3.4**).

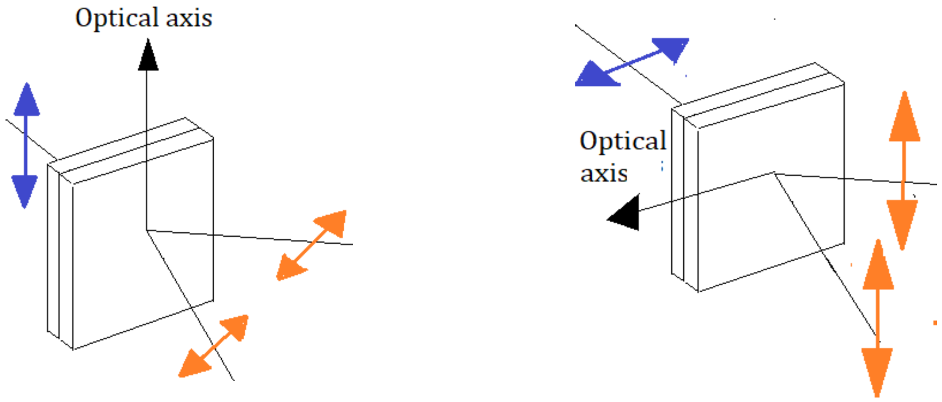

Figure 3.4. Outgoing down-converted photons for vertical and horizontal polarization input.

If the optical axis of the crystal is placed vertically and the incoming photons are vertically polarized too then the crystal splits them into two horizontally polarized photons. On the contrary, if the optical axis of the crystal is placed horizontally and the input photons are horizontally polarized then, at the output, two vertically polarized photons will be generated (see [31]).

These crystals have two indices of refraction commonly known as extraordinary and ordinary. There are two types of down-conversion according to them named type-I and type-II. In order to obtain entangled photons it is necessary that the input polarization is neither horizontal nor vertical but diagonal, typically 45 degrees. The impact a diagonal polarization will have when passing through the crystals is that the outgoing photons will have an indeterminate polarization because as both crystals are thin and are very close to each other that will cause a quantum incertitude about which crystal the photon will interact with and, as a consequence, what the output polarization will be. In type-I SPDC both outgoing photons are ordinary which means they will have the same polarization; both generated photons will be either horizontal or vertical (**3.5**). When measuring one of the entangled photons at the output the other will automatically collapse and reveal the same polarization state. On the other hand, for type-II down-conversion, the outgoing photons will be one of each kind: one ordinary and one extraordinary; they will be orthogonal. The signal will be horizontal and the idler, vertical and vice versa (**3.6**) which means that when measuring one of the entangled outgoing photons the other will immediately collapse revealing the opposite polarization.

$$|\psi\rangle = |HH\rangle + |VV\rangle \tag{3.5}$$

$$|\psi\rangle = |HV\rangle + |HV\rangle \tag{3.6}$$

It is important to have into consideration that in order to achieve down-conversion type-I two BBO crystals will be needed while for type-II down-conversion only one crystal will suffice. The relationship of correlation that exists between the output down-converted photons makes the production of entangled photons possible. When two photons are in a correlated polarization state, measuring the polarization of one will set the state of the other without having to measure it. SPDC is the proper procedure to obtain photons that are both entangled.

For type-I down-conversion the output has the shape of two concentric cones and the entangled photons are placed around the cones, opposite to each other (see **Figure 3.5**).

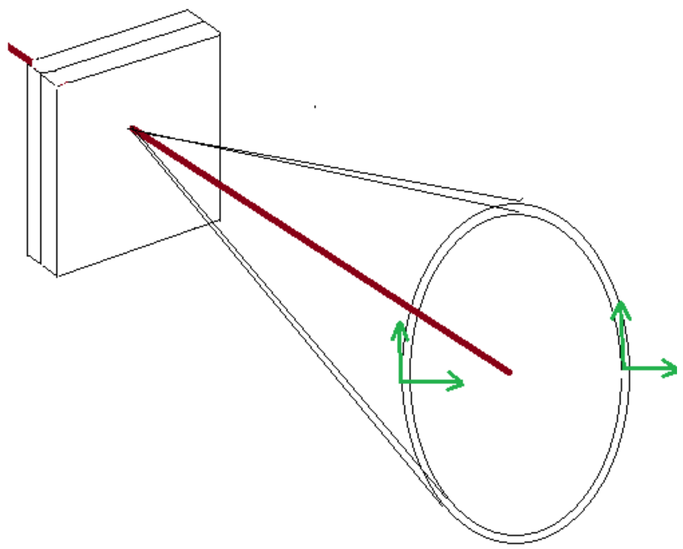

Figure 3.5. Type-I entangled photons. The crystals are too close one from the other to determine which crystal the photons originated from.

In case of SPDC type-II the pump beam is converted into two cones. The outgoing entangled photons will be found in the place where the cones cut each other because the polarization at these two points is unknown **(Figure 3.6).**

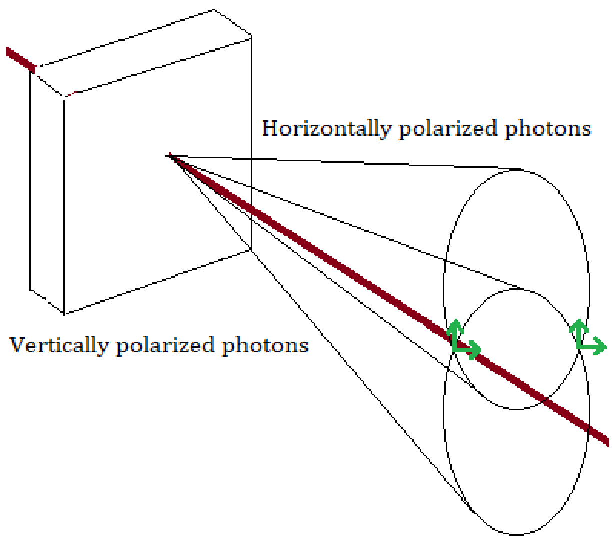

Figure 3.6. Type-II entangled photons.

In order to produce entangled photons there is a mandatory requirement: the input polarization has to be indeterminate. Not horizontal, not vertical, not circular. Then the input polarization must have both vertical and horizontal polarization so that entanglement takes place. 45 degrees linear polarization is quite recommendable as it has components in both directions, the horizontal and the vertical. If this requirement is fulfilled then two entangled photons will be generated.

Now let us focus in obtaining type-I SPDC. The first requirement to obtain it is that one of the crystals must be oriented vertically – optical axis placed verticaly – while the other must be horizontally oriented. If this condition is met then when the photon reaches the crystals an uncertainty about with which crystal will the photon interact will exist and, therefore, what polarization the outgoing photons will have. Their state will only be determined once one of them is measured.

The image below is a representation of a BBO and the angles in its interior. The expression **(3.7)** is the condition that must be fulfilled so that type-I SPDC is achieved.

$$n_p = n_s \cos \theta_c \qquad \textbf{(3.7)}$$

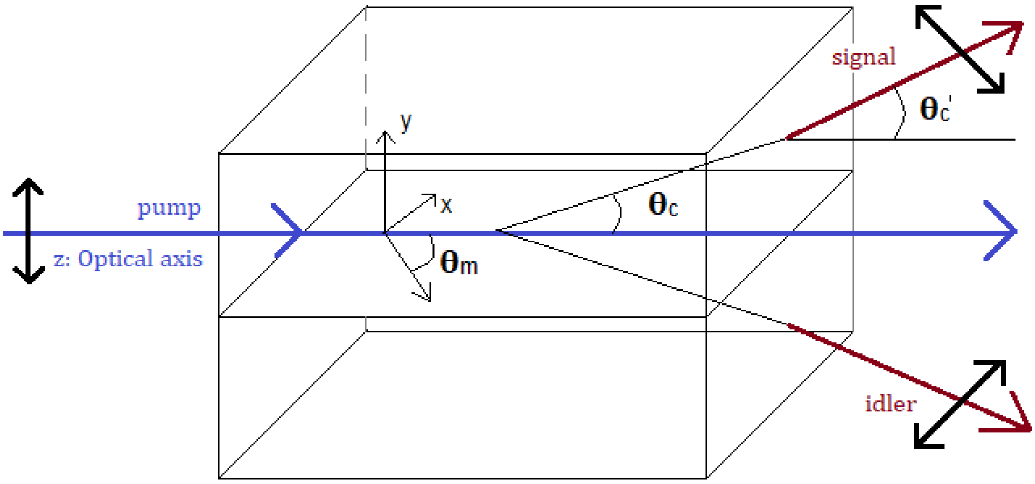

Figure 3.7. Angles inside BBO crystal

θ$_c$ is the angle the signal and idler photons form with the direction of propagation of the pump inside the crystal, n_s the index of refraction of the pump beam and n_s the index of refraction of the signal photon. The polarization is in the same plane as the optical axis the index of refraction depends on the angle according to the following relationship.

$$\tilde{n}_e(\theta_m) = (\frac{cos^2\theta_m}{n_o^2} + \frac{sin^2\theta_m}{n_e^2})^{-1/2} \qquad \textbf{(3.8)}$$

Where n_o and n_e are the ordinary and extraordinary indices of refraction of the crystal respectively.

3.3 Polarizers

These elements are specifically designed to change or measure the polarization of incident light at different ranges of frequencies as UV (100 – 400 nm), IR (>780 nm) and visible (400 – 780 nm). Polarizers' main characteristic is determining the state of polarization of the beam of light at the input.

In the next subsection the most used polarizers will be defined together with a description of their performance.

3.3.1 Wave Plates

Optical wave plates are made of birefringent materials. Birefringence is the property of non-isotropic transparent materials for which the refractive index depends on the direction of the polarization. These materials have different properties depending on the direction of the incident light. This property is observed for crystalline quartz, calcite, sapphire, ruby and also for nonlinear crystal materials.

The wave plates that will be considered in this work are made of quartz and exhibit uniaxial anisotropy. This means that there is a single direction governing the optical anisotropy whereas all directions perpendicular to it are optically equivalent. As a consequence we will have two orthogonal axes corresponding to each refractive index. These axes are called fast and slow. The fast principal axis of the wave plate has a low refractive index which generates light at a fast speed when it is polarized according to this direction. On the contrary, the slow axis has a high refractive index giving as a result light at a slow speed at the output. When light passes through a wave plate the differences in speed lead to a phase mismatch between the two orthogonal components of polarization. This phase shift depends on the properties of the material, the thickness of the wave plate and the wavelength of the light.

The most common wave plates are Half Wave Plates (HWP) and Quarter Wave Plates (QWP) for which the difference of phase delays between the two linear polarization directions is π and $\frac{\pi}{2}$ respectively. Their main characteristic is that they are able to alter the polarization state of an incident beam because they are based on the birefringence principle.

Half wave plates add a phase of 90 degrees to any incoming horizontal or vertical polarization. If the input polarization if vertical the output will be horizontal and vice versa. In the case of a circular input the HWP will change its sense of rotation (Figure 3.5):

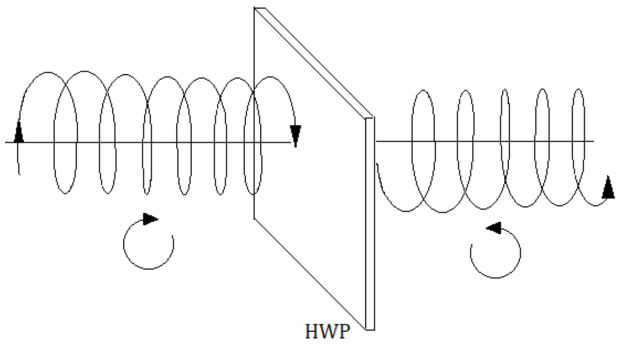

Figure 3.8. Behavior of a HWP for an incoming circular polarization. The half wave plate rotates its sense of rotation.

A QWP turns linear polarization into circular polarization and vice versa, circular polarization into linear polarization (see figures 3.8 and 3.9).

As already mentioned, the quarter wave plate introduces a phase shift of $\frac{\pi}{2}$ between the two components of the incoming polarization.

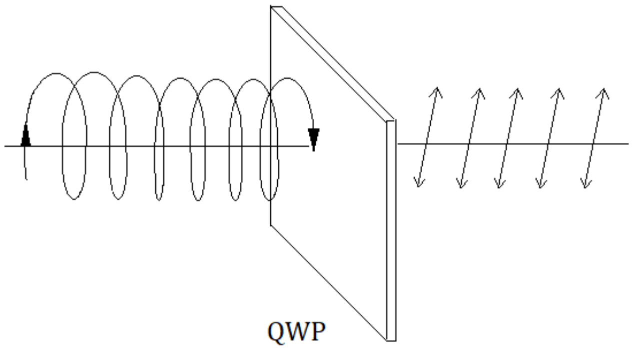

Figure 3.8. Performance of a quarter wave plate for a circular input.

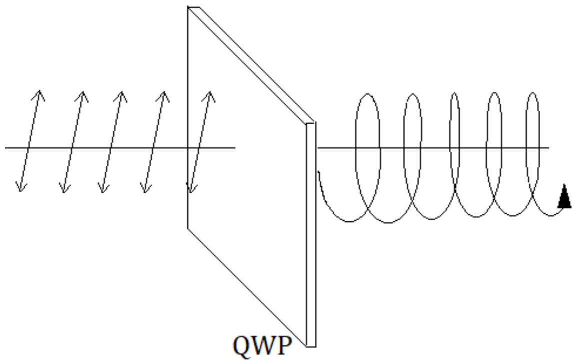

Figure 3.9. Performance of a quarter wave plate for a linear input.

3.4 Detectors: Avalanche Photodiodes (APD)

Avalanche photodiodes are highly sensitive and high speed semi-conductor devices whose function is transforming light into electricity. They are equipped with an internal region where electron multiplication occurs by means of an applied external voltage. The most common APDs are fabricated from silicon materials and show a precise sensitivity in the 450 to 1000 nm wavelength interval (see [33]).

The most common application of APDs is the measurement of quantum efficiency which means that, from the measures of these devices, one can know the level of absorption of photons. In this work APDs will be used as single photon counting modules (SPCM). SPCMs have its highest efficiency at a wavelength of 700 nm so the ideal source of light would be a laser with a pump beam at 350 nm but laser that operate at this wavelength are very expensive and hard to find. Then other option is using a laser of a wavelength of 405 nm because when generating down-converted photons at 810 nm SPCMs will offer a detection efficiency of 60%.

CHAPTER 4. QUANTUM OPTICAL SETUPS

In this Chapter a feasibility study for setting up a quantum optical laboratory for undergraduate students is presented. The main objective, as stated in the introduction, is to provide EETAC School with a practice laboratory giving certain visibility of the acquired knowledgment during TIQ lectures as well as to be an element for introductory studies in the quantum communication field. Consequently, the main idea is to dig into all the possibilities for analyzing what would suit better our needs taking into account the available budget or the level of complexity of the experiments to implement. Optical elements are used for specialized disciplines and have many different properties that make them expensive. Of course, the price depends on many factors and there are optical apparatus cheaper than others. This is the reason why a first study on the different options in the market was done in order to estimate how much the assembling of a so called quantum optical laboratory would suppose.

4.1 Educational kits

Before starting to describe the elements that compose the quantum entanglement scenario it is worth saying that some previous research was done on the field in order to see what the offer from different providers was and how many options could we count on. During this phase of research the American company Thorlabs (see [34]) presented some interesting educational products and kits on its catalogue of products whose main objective is promoting optics among students through different experiments. After examining the different kits, the conclusion was that two of them were suitable. These kits are the Quantum Cryptography Kit and the Quantum Eraser Demonstration Kit.

The idea is that the combination of these kits with other optical elements that can be bought separately the setup for the implementation of more advanced experiment can be constructed (adding or removing optical elements).

4.2 BB84 Protocol

The first kit allows implementing an experiment of quantum cryptography by simulating the encryption, transmission and decryption of a secret message. It also shows how a third party can interfere the communication channel and how to detect it. The protocol of communication to be performed with this kit is the so called BB84. This protocol was first outlined in 1984 by Charles H. Bennett and Gilles Brassard and was the first quantum cryptography protocol based on quantum mechanics. The principal actors are Alice and Bob and play their roles of sender and receiver. They are connected by a physical channel through which polarized light will be sent between them. For ensuring the secure transmission of information Alice will have to choose between two different bases – which can be translated into 4 different polarizations – for representing 0 and 1. On the other hand, Bob will also choose one out of two basis for measuring the incoming polarizations sent by Alice: if Alice and Bob are using the same basis Bob's measure will coincide with Alice's but if he chooses a different basis he will obtain no information on the state Alice sent. They share the bases they used via telephone, for instance. This will let them extract the sifted key that will allow them to obtain the secret key later. The measure of the error rate would alert them of the presence of an evesdropper (see [47]). The setup for carrying out this experiment is shown in **Figure 4.1**.

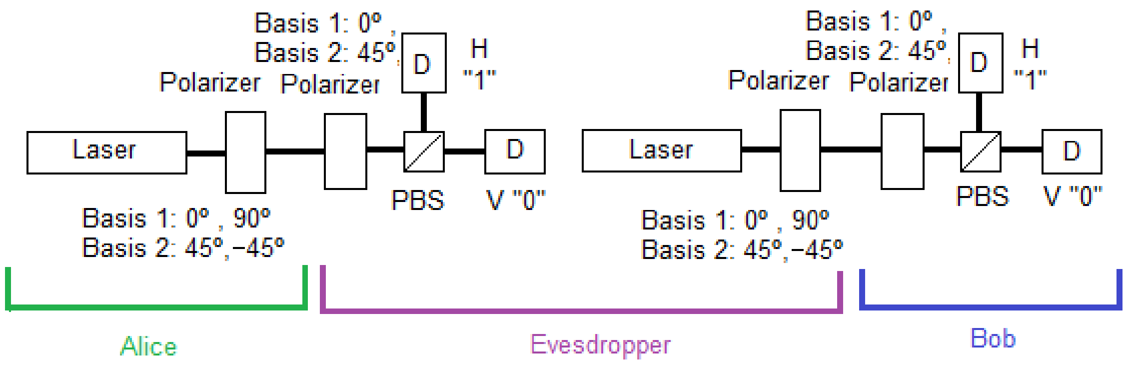

Figure 4.1. BB84 Protocol

For eavesdropping Eve implements both parts – Alice's and Bob's – but in reverse order. The initials PBS stand for Polarizing Beam Splitter whose function, as it name implies, is splitting or dividing the different polarizations at the input through two different paths according to the polarization. Beam splitters are one side coated which means that there is a thin layer of a reflective dielectric material responsible of introducing a phase shift of 0 or π depending on the side where the light collides with the crystal. These two paths are called transmitted and reflected **(Figure 4.2)** and, usually, vertical polarization is transmitted while horizontal is reflected. The most common form for this optical device is a cube composed by two triangular prisms attached together (see [48]). The rule is that transmitted light has no phase shift while a phase of π is added to reflected light depending if the side of the crystal where light collides is coated.

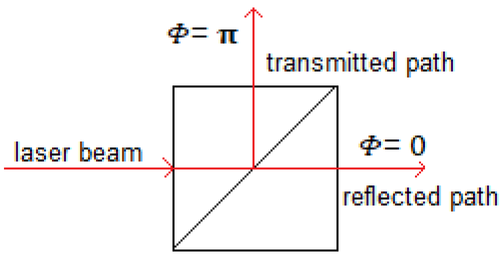

Figure 4.2. Diagram of the Polarizing Beam Splitter

Any input state will be reflected and transmitted adding a phase of 90 degrees to the reflected (vertical) path. All the vertical photons at the input will be transmitted whereas all the horizontal photons will be reflected.

Bob will only need to set his polarizer to 0º or 45º for measuring the information sent by Alice. Finally, the initial D in **Figure 4.1** stands for detector. In the case of the kit the detectors supplied are just sensors that will activate when they detect the presence of a photon. As no further specifications on these sensors are provided by the vendor it will be necessary to acquire APDs for performing more advanced experiments because precise measurements are required.

4.3 Entanglement experiment

The first step is defining which experiment is the most advantageous and profitable for observing quantum optical effects. As already introduced during the previous chapters entanglement is a quite interesting quantum phenomenon which is on the basis of any quantum computation application such as teleportation, superdense coding, or quantum cryptography which is, at the same time, the basis for setting up secure communication channels. The setup for an entanglement experiment is as follows:

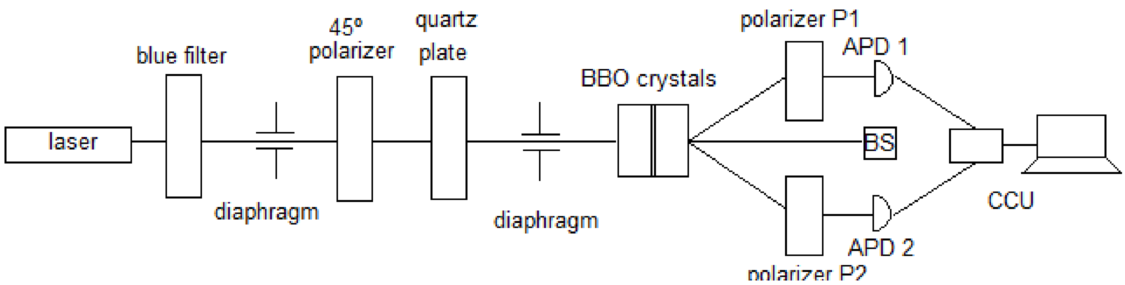

Figure 4.3. Entanglement setup

As shown in Figure 4.1 the source of light is a laser. Thorlabs offers different types of laser sets depending on the range of frequencies, in this case, between 350 and 700 nm. The laser has a wavelength of 405 nm which is visible blue light and some of the most commercially common lasers for many applications such as telecommunications, medical diagnostics and information technology. It is important to keep in mind that the use of secure equipment as well as safe practices should be taken into consideration when operating this devices. For avoiding any unexpected incident the use of secure eyewear is highly recommended as metallic tools in the laboratory could redirect the laser beam.

With the objective of exploring different options and prices, two different possible lasers are provided so that the pros and cons are evaluated and finally one of them chosen. The decision will depend of course of the needs to cover and on the available budget.

The first option is a using a diode. In this case, the chosen laser provides a range of wavelengths between 400 and 410 nm whose center wavelength is 405 nm. This wavelength varies from a run to another but it can be tuned with the temperature. In order to work with the diode other components are needed. Fortunately the vendor (Thorlabs) provides a complete kit with both diode and temperature controllers. The kit also includes the mount, optical components and accessories. The controllers work at an adjustable line voltage and offer constant currents and power to drive the diode in order to ensure outstanding performance. In particular, the temperature controller is precise allowing the stabilization of laser diodes and the control of the temperature. The mount included in the kit will control the laser diode and monitor the photodiode. Since the output of the laser diode is quite divergent, collimation is necessary. Aspheric lenses are a great help for narrowing light beams and are already included in the kit.

Another laser option is the one provided by CrystaLaser (see [35]) for which a quotation was requested. In this case the laser provides the same range of operation wavelengths – between 400 and 410 nm – as the Thorlabs option. The laser is also collimated and was quoted including the corresponding power supply and, in this case, there is no need for a temperature controller module as it is stabilized internally.

In the table below we summarize the main features of both lasers. For further details on the characteristics of the lasers please refer to the specifications sheets.

Table 4.1. Laser specifications of our two models analyzed in this work.

	Thorlabs L405P20	CrystaLaser DL405-050-O	Unit
Range of wavelengths	400 - 405	400 - 405	nm
Typical wavelength	405	405	nm
Optical output power	35	50	mW
Power supply	Included	Included	N/A
Temperature controller	Included	N/A	N/A
Specifications	See [36]	See [37]	N/A

The laser will emit an unpolarized blue light that will pass through a band pass filter and an aperture with the objective of narrowing the light beam and avoiding any dispersion of its spectrum. The aperture consists of a sheet of aluminum with a small hole previously scooped out in it (see [38]). There are other more sophisticated options available as for example using a diaphragm.

Once the light has been filtered and has passed the aperture the next optical element it will collide with is the polarizer. This polarizer will be the responsible of setting the polarization to the required direction. As previously explained (see 3.3. BBO) the polarizer should be set for the light at the output to have a linear polarization of 45 degrees. As the polarizer is unmounted a mounting cage will be needed. Mounting cages are really useful as they allow the integration of optical crystals and its manipulation since a rotation of 360 degrees is allowed so it will only be necessary to adjust the mount to 45 degrees for obtaining the polarization we need. The resulting intensity after crossing a polarizer is half of the unpolarized initial intensity.

The next element in the path is a quartz plate responsible of compensating the phase shift introduced by the BBO crystals. In order to get the quartz plate to compensate this phase shift certain orientation will be needed; this orientation will be achieved by doing different measurements with different orientation angles. BBO crystals are not-ideal birefringent materials, so they can introduce dispersion which can be translated into a phase imbalance of the generated pair of photons. This is a consequence of using two crystals, so photons will be generated at different points in space making the outgoing entangled photons present a mismatch between their phases that will need to be previously compensated (see [40]).

Once this phase mismatch compensated the light will reach our type-I BBO crystals. In this case the provider is Eksma Optics, a global manufacturer of optical components and systems for many optical applications. They offer a complete catalogue of nonlinear optical crystals.

In order to get type-I down-conversion the phase matching condition (3.15) must be met. The outgoing photons will be orthogonal to the polarization of the pump photons but the polarization of both of them will be the same. The election of the crystal is determined by the phase matching angle θ_m and the down-converted photons will be generated at $\lambda_s = \lambda_i = 810\ nm$. These photons will be affected by the ordinary index of refraction of the crystal denoted by $\tilde{n}(\theta_m)$ and represented as follows (see [41]):

$$\tilde{n}(\theta_m, \lambda) = \left(\frac{\cos(\theta_m)^2}{n_o(\lambda)^2} + \frac{\sin(\theta_m)^2}{n_e\ (\lambda)^2}\right)^{-1/2} \quad \textbf{(4.1)}$$

The following expressions should be also considered (see [41]):

$$n_s = n_o(\lambda_s) \quad \textbf{(4.2)}$$

$$n_p = \tilde{n}\big(\theta_m, \lambda_p\big) \quad \textbf{(4.3)}$$

Then, knowing **(3.15)** and using **(4.2)** and **(4.3)** we get the following equality (see [41]):

$$\tilde{n}\left(\theta_m, \lambda_p\right) = n_o(\lambda_s) \cos\theta_c \quad \textbf{(4.4)}$$

If our pump beam's wavelength is $\lambda_p = 405\,nm$, the signal and idler are placed at $\lambda_s = 810\,nm$ and if we set the divergence of the outgoing photons to approximately 3 degrees – which is almost the walk-off angle of the crystals – we will know the phase matching angle that will let us choose the corresponding crystals (see [41]):

$$\left(\frac{\cos(\theta_m)^2}{n_o\left(\lambda_p\right)^2} + \frac{\sin(\theta_m)^2}{n_e\left(\lambda_p\right)^2}\right)^{-1/2} = n_o(\lambda_s) \cos\theta_c \quad \textbf{(4.5)}$$

At this point one could have realized that in order to proceed with the calculations it is absolutely necessary to know how to find n_o and n_e. The way to find them is by means of the Sellmeier equations. They are a set of two expressions that relate the index of refraction *n* with the wavelength of a certain transparent medium. These expressions are given below (see [42]):

$$n_o{}^2 = 2.7366122 + \frac{0.0185720}{\lambda^2 - 0.0178746} - 0.0143756\lambda^2 \quad \textbf{(4.6)}$$

$$n_e{}^2 = 2.3698703 + \frac{0.0128445}{\lambda^2 - 0.0153064} - 0.00153064\lambda^2 \quad \textbf{(4.7)}$$

It is important to notice that when replacing values the wavelength must be expressed in micrometers (μm) instead of nanometers (nm). The obtained results are listed next:

$$n_o\left(\lambda_p = 0.405\right)^2 = 1.69154628 \quad \textbf{(4.8)}$$

$$n_e\left(\lambda_p = 0.405\right)^2 = 1.56708657 \quad \textbf{(4.9)}$$

$$n_o(\lambda_s = 0.810)^2 = 1.660204748 \quad \textbf{(4.10)}$$

Now, replacing these values in **(4.5)** we get

$$\left(\frac{\cos(\theta_m)^2}{1.69154628^2}+\frac{\sin(\theta_m)^2}{1.567086657^2}\right)^{-1/2} = 1.660204748\cos 3° \qquad \textbf{(4.11)}$$

At this stage the way to find the phase matching angle for choosing the BBO crystals is solving **(4.9)** and determining θ_m.Once done the final result is 29.2° and allows us to filter the different BBO crystals in the Eksma Optics' website.

When working with parametric conversion it is important to have into account that just one out of a billion pump photons will actually interact inside the crystal and divide, the others will continue their way out through the crystal. It is really important to block the pump photons that cross the BBOs so that they do not get mixed up with the signal and the idler (see [43]). With the objective of blocking these pump photons a beam stop will be needed. Beam stops or beam dumps are optical elements whose function is, as their name implies, absorbing or containing the advance of photons. For systems with a low output power this device can be replaced by a piece of black velvet but more sophisticated or advanced systems require more powerful devices for retaining outgoing pump photons.

Once the light crosses the BBO crystals, type-I down-converted photons is likely to have taken place. To highlight that the outgoing photons are entangled we will need a system that lets us measure coincidences The elements that will help us with these measurements are APDs or avalanche photodiodes already introduced in this thesis in section **3.4**. Before reaching the APDs, the generated signal and the idler photon will pass each through a polarizer. Then there are four possibilities likely to happen: as the two outgoing photons are assumed to be parallel if the polarizers are both set to be parallel to each other then, there will be coincidence. If they are set to be orthogonal polarized to one another there will be no simultaneous detection. There is also the possibility of the generated photons not to be entangled. In this case when the polarizers are set parallel to each other there will be a probability of 50% of detection for each one which leads us to a 25% of coincidence of the system. Finally, if the polarizers are set to be orthogonal to each other the probability of coincidence will be also a 25%. The table below summarizes these four possibilities:

Table 4.2. Summary of coincidence detection for polarizer P1 and P2.

Photon pairs	Polarizers	Coincidence
Entangled	Parallel	Yes
	Orthogonal	No
Not entangled	Parallel	$0.5\cdot 0.5 = 0.25 = 25\%$
	Orthogonal	$0.5\cdot 0.5 = 0.25 = 25\%$

It is worth saying that the polarizers will set the polarization of the signal and the idler that will be generated in an uncertain state. The way to determine coincidence is using the APDs and a coincidence detector. The cheapest way of arranging such detector implies a bit of knowledge in electronics as it consists in assembling an electric circuit composed by some flip flops, capacitors and resistors (see [44]). There are other options but they are more expensive. Nevertheless, they should also be taken into account. Altera DE2 Board is a well known FPGA electronic board oriented to development with educational purposes as it provides ways for programming prototypes. DE2 allows to easily program a CCU (Coincidence Counting Unit) for detecting coincidences (see [45]). Once the software code is developed the way for counting coincidences is doing it through P1 and P2 the following way: P1 will be fixed while P2 will be the polarizer to rotate. There will be, for example, 4 different scenarios: P1 will be fixed to an angle of 0 degrees while P2 will do an entire rotation moving from 0 to 360 degrees in steps of 10 degrees. The procedure for the next scenario is the same but setting P1 to 45 degrees and rotating P2. For configuring the third and fourth scenarios P1 will be set to 90 and 135 degrees while P2 will rotate as for scenarios one and two. The measurements taken will show a response with the shape of a cosine squared function for each scenario. The results will be represented in a chart where the x-axis stand for coincidence photon counts while y-axis will picture the steps taken during the rotation of polarizer number two (from 0 to 360 degrees) (see [46]). The result will be a graph where the number of counts is represented with reference to the angle set to P2; it will be an oscillating positive curve as shown in **Figure 4.4.**

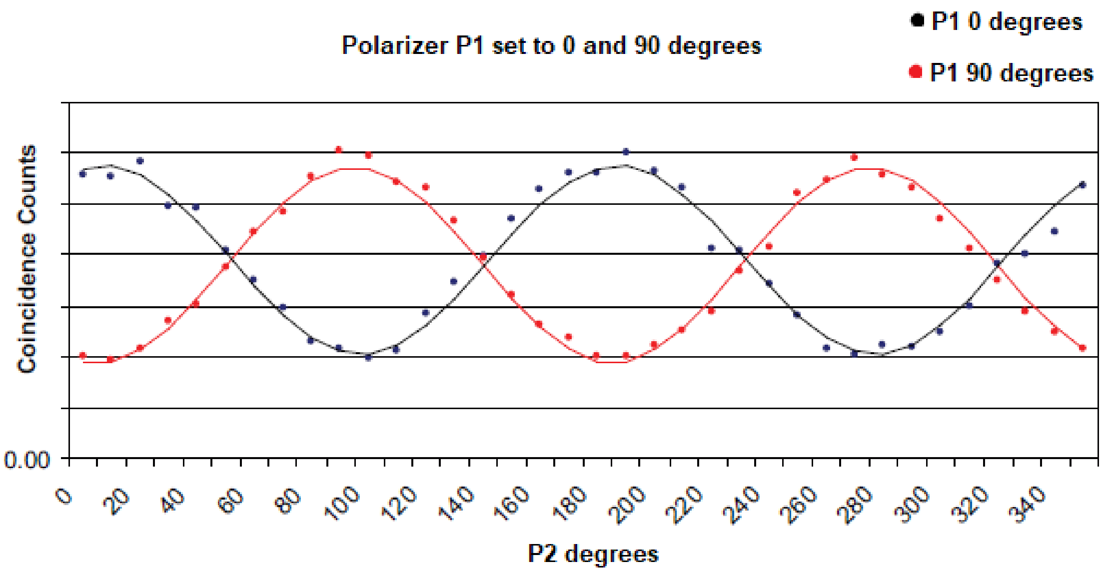

Figure 4.4. Representation of the oscillating positive curve obtained when counting coincidences at the APDs.

After passing the polarizers P1 and P2 the photons will also come across a diaphragm and a red filter that will help sharpen the beam of light and avoid any dispersion that may have been generated from propagation and finally a focusing lens in order to focus light into the APDs. So, in order to place the detectors at the right distance it is important to first calculate the focal length of the lenses. For doing it, it is necessary to know the beam divergence and the desired beam diameter. In order to collect as much light as possible during this process the divergence angle of the laser to be taken into account is the largest in the specifications sheets (19 degrees).

The Quantum Cryptography Kit includes a laser of wavelengt 635 nm. It would be interesting to know if we could also make use of this laser for implementing entanglement with the same BBO crystals. The way to operate would be the same as already done for the case of 405 nm laser in **(4.6)**, **(4.7), (4.8)** and **(4.9).** First, using Sellmeier equations the ordinary and extraordinary indices will be found:

$$n_o\left(\lambda_p = 0.635\right)^2 = 1{,}667036501 \quad \textbf{(4.12)}$$

$$n_e\left(\lambda_p = 0.635\right)^2 = 1{,}54977646 \quad \textbf{(4.13)}$$

In this case we will assume that we want down-converted photons at 810 nm too:

$$n_o(\lambda_s = 0.810)^2 = 1.660204748 \quad \textbf{(4.14)}$$

Now, replacing these values in **(4.5)** we get

$$\left(\frac{\cos(29.2°)^2}{1{,}667036501^2} + \frac{\sin(29.2°)^2}{1{,}54977646^2}\right)^{-1/2} = 1.660204748\cos\alpha \quad \textbf{(4.15)}$$

Replacing these values in **(4.5)** we obtain that, in this case, α = 1.92°. This angle represents the divergence of the down-converted photons with respect to the pump beam.

BBO crystals present what is called a walk-off angle. This parameter indicates the maximum angle for which the output beam will drift away from the pump beam. For the crystals chosen this angle is of 3.19° as per supplier specifications. Then, the maximum frequency at which signal and idler could be generated for both input wavelengths of the lasers is found bellow.

$$\lambda_p = 405\ nm$$

$$\left(\frac{\cos(29.2°)^2}{1.69154628} + \frac{\sin(29.2°)^2}{1.56708657}\right)^{-1/2} = n_o(\lambda_s)\cos 3.19° \qquad \textbf{(4.16)}$$

➔$\lambda_{s\_max} \backsimeq 810\ nm$

$$\lambda_p = 635\ nm$$

$$\left(\frac{\cos(29.2°)^2}{1{,}667036501^2} + \frac{\sin(29.2°)^2}{1{,}54977646^2}\right)^{-1/2} = n_o(\lambda_s)\cos 3.19° \qquad \textbf{(4.17)}$$

➔ $\lambda_{s\_max} \backsimeq 1790\ nm$

Taking into account the calculations in **(4.15)** and **(4.17)** we can conclude that the laser of 635 nm included in the kit would also be useful for our entanglement experiment since the angle of divergence of the down-converted photons is smaller than the walk-off angle and the maximum wavelength for obtaining down-conversion is 1790 nm.

CHAPTER 5. PRICELISTS FOR UNDERGRADUATE QUANTUM COMMUNICATION EXPERIMENTS

This chapter aims to detail the elements that will be needed for the construction of the experiments described along the previous chapters. In order to have wider overview of the range of prices different alternatives have been taken into account and different providers have been checked with the objective of comparing different options and choosing the most affordable providers. The idea is having different pricelists by removing and adding elements for analyzing the difference on the price later.

5.1 Quantum Cryptography experiment

The quantum kit to perform this experiment is already available at Thorlabs.com in two versions depending on the units of measurement. In our case the metric version is the most suitable. **Table 5.1** gathers all the elements that will be included in the kit. The Quantum Cryptography Kit costs 2985,00 €. There are some optical apparatus included in the kit that are not sold separately; they are indicated with NA. The total price corresponds to the elements when bought separately.

Table 5.1. Quantum Cryptography kit elements, quantities and prices

Optical element	Quantity	Price per unit (€)	Total (€)
635 nm laser diode module	2	NA	NA
Laser electronics module	2	NA	NA
Half wave plate	2	246,00	492,00
Polarizing Beam Splitter	2	184,00	368,00
Sensor	2	NA	NA
Sensor electronics module	2	NA	NA
Kinematic mount	2	34,83	69,66
Collimator mounting adapter	2	20,40	40,80
Breadboard feet (4 units)	3	4,73	14,19
Breadboard handles (2 units)	1	13,30	13,30
Spanner wrench	1	24,20	24,20
Kinematic platform mount	2	NA	NA
Variable Height Clamp	2	4,59	9,18
Aligment tool	1	20,40	20,40
Rotating post holder base	1	4,68	4,68
Clamping arm	2	16,70	33,40

Rotation mount	4	NA	NA
Optical post (50 mm)	7	4,67	32,69
Optical post (40 mm)	2	4,47	8,94
Optical post (30 mm)	4	4,27	17,08
Post holder (50 mm)	5	6,93	34,65
Post holder (40 mm)	6	6,50	39,00
Post holder base	10	5,04	50,40
Breadboard (20 cm x 20 cm)	1	66,75	66,75
Breadboard (20 cm x 25 cm)	1	102,00	102,00
Breadboard (30 cm x 45 cm)	1	171,00	171,00
Balldriver (6 mm)	1	7,65	7,65
Hex key (3 mm)	1	NA	NA
Hex key (2 mm)	1	NA	NA
Hex key (1,5 mm)	1	NA	NA
Washer (100 units)	19	4,18	79,42
Screw (45 mm)	2	NA	NA
Screw (30 mm)	2	NA	NA
Screw (16 mm - 25 units)	17	7,45	126,65
Screw (12 mm - 25 units)	12	7,20	86,40
Screw (10 mm - 25 units)	11	7,00	77,00
Screw (30 mm)	2	NA	NA
Total (€) - Elements bought separately. NA not included	1989,44		
Kit total Price (€)	2985,00		

Table 5.2 shows the breakdown of prices of the elements indicated as NA in **Table 5.1.** Taking this priceslist into account the final price for the elements bought separately is 3145 € but sensors and sensor electronics are not even included.

Table 5.2. Quantum Cryptography kit elements bought separately

Optical element	Reference	Quantity	Price per unit (€)	Total (€)
635 nm laser diode module	CPS405	2	81,75	163,50
Laser electronics module	KAD11NT	2	55,75	111,50
Fixed optical mount	FMP1	2	14,40	28,80
Sensor	-	2	-	-
Sensor electronics module	-	2	-	-
Kinematic platform mount	KM100PM	2	67,25	134,50
Rotation mount	RSP1X225	4	119,00	476,00

Hex key (3 mm)	HKM50NM	1	52,72	52,72
Hex key (2 mm)	HKM20NM	1	42,72	42,72
Hex key (1,5 mm)	HKM15NM	1	42,72	42,72
Hardware kit	HW-KIT2/M M6	1	102,00	102,00
Total (€)	1156,46			

As a conclusion it is worth saying that purchasing Thorlabs' kit is more than advantageous as it implies saving money and also the effort of searching for suitable sensors on other providers' catalogue.

5.2 Quantum Entanglement experiment

In the case of the Quantum Entanglement experiment a pricelist with multiple options is shown, since there are different providers for some of the elements to use. For instance, we have different options for the laser or BBO crystals, as already explained in previous chapters.

5.2.1 Option 1: Thorlabs laser

This pricelist details the required elements for carrying out the Entanglement experiment using a laser diode provided by Thorlabs. In order to have the laser running both a current controller and a temperature controller are needed as well as a laser mount that will help stabilize the laser. SCPMs are, without doubt, the elements that will shoot up the price. They are high efficiency detectors since they keep the coincidences rate at a good level. The provider we chose for these elements is Excelitas. These modules offer an actual count rate of photons of 27 million of counts per second according to the actual count rate of photons provided by Excelitas:

$$Actual\ count\ rate\ of\ photons = \frac{a \cdot b - c}{d} = \frac{27Mc}{s} \quad \textbf{(5.1)}$$

$$Correction\ factor = \frac{1}{1-(t_d - C_R)} = 1{,}36 \quad \textbf{(5.2)}$$

In **(5.1)** a is the output module count rate, b the correction factor at the module count rate, c the dark count module rate and d the photon detection efficiency. Whereas, in **(5.2)**, t_d is the module dead time and C_R the output count rate. All these parameters are available at SCPMs datasheets.

Table 5.3. Quantum Entanglement experiment elements, quantities and prices

Optical element	Reference	Provider	Quantity	Price per unit (€)	Total (€)
405 laser diode	L405P20	Thorlabs	1	44,50	44,50
Current controller	LDC205C	Thorlabs	1	917,00	917,00
Temperature controller	TED200C	Thorlabs	1	898,00	898,00
Laser diode mount	TCLDM9	Thorlabs	1	446,00	446,00
Spanner wrench	SPW301	Thorlabs	1	13,00	13,00
Adapters	SPW909	Thorlabs	1	26,50	26,50
Post	TR3	Thorlabs	1	4,88	4,88
Mounting base	BA2	Thorlabs	1	6,57	6,57
Optic adapter	S1TM09	Thorlabs	1	20,40	20,40
Locking nut	SM1NT	Thorlabs	1	6,05	6,05
Collimation lens	C230TMD-A	Thorlabs	1	61,50	61,50
Mounted Bandpass Filter	FGB37M	Thorlabs	1	58,50	58,50
Mount for optics	TRF90/M	Thorlabs	1	74,50	74,50
Post holder	PH75/M	Thorlabs	7	7,44	52,08
Optical post	TR75/M	Thorlabs	7	4,88	34,16
Screw (25 units)	SH6MS10	Thorlabs	3	7,00	21,00
Mounted iris (diaphragm)	ID8/M	Thorlabs	4	42,75	171,00
Linear Polarizer	LPVISE200-A	Thorlabs	3	82,75	248,25
Rotation mount	RSP1X15(/M)	Thorlabs	3	119,00	357,00
Clamping fork	CF125	Thorlabs	7	9,75	68,25
Base adapter	BE1	Thorlabs	7	8,45	59,15
Beam trap	BT610(/M)	Thorlabs	1	269,00	269,00
BBO crystals	BBO-604H	Eksma Optics	2	390,00	780,00
Rod holder base adapter	820-0225	Eksma Optics	1	6,00	6,00
Standard rod holder	820-0051	Eksma Optics	1	12,50	12,50
Hex key (10 pieces)	820-0270	Eksma Optics	1	7,30	7,30
Screw (10 pieces)	820-0560	Eksma Optics	1	7,30	7,30
Longpass filter (800 nm)	#66-059	Edmund Optics	2	28,50	57,00
Aespheric lenses		Thorlabs	2	60,00	120,00
Altera DE2 board (Academic discount applied)	-	Terasic	1	253,78	253,78

Single photon counting module	SPCM-AQRH-11	Excelitas	2	3022,00	6044,00
Total (€)	11144,00				

Additionally, the optical wrench for assembling this experiment can be taken from Thorlabs kits as well as rotation mounts, screws, adapters and clamping forks.

Finally, regarding Altera DE2 board, the provider Terasic offers an academic discount of 188€ that has already been applied for calculating the final price.

5.2.2 Option 2: Crystalaser laser

This option details also the prices for carrying out the experiment of entanglement but considers another provider for the source of light. As the prices for the laser were not available in CrystaLaser website a quotation was required. The laser has a wavelength of 405 nm and 50 mW of power. As well as Thorlabs laser, this one also needs a power supply for operating and was already included in the quotation.

Table 5.4. Quantum Entanglement experiment elements, quantities and prices

Optical element	Reference	Provider	Quantity	Price per unit (€)	Total (€)
Laser	DL405-050-O	CrystaLaser	1	2680,00	2680,00
Power supply			1		
Spanner wrench	SPW301	Thorlabs	1	13,00	13,00
Optic adapter	S1TM09	Thorlabs	1	20,40	20,40
Locking nut	SM1NT	Thorlabs	1	6,05	6,05
Collimation lens	C230TMD-A	Thorlabs	1	61,50	61,50
Mounted Bandpass Filter	FGB37M	Thorlabs	1	58,50	58,50
Mount for optics	TRF90/M	Thorlabs	1	74,50	74,50
Post holder	PH75/M	Thorlabs	6	7,44	44,64
Optical post	TR75/M	Thorlabs	6	4,88	29,28
Screw (25 units)	SH6MS10	Thorlabs	1	7,00	7,00
Mounted iris (diaphragm)	ID8/M	Thorlabs	4	42,75	171,00

Linear Polarizer	LPVISE200-A	Thorlabs	3	82,75	248,25
Rotation mount	RSP1X15(/M)	Thorlabs	3	119,00	357,00
Clamping fork	CF125	Thorlabs	6	9,75	58,50
Base adapter	BE1	Thorlabs	6	8,45	50,70
Beam trap	BT610(/M)	Thorlabs	1	269,00	269,00
BBO crystals	BBO-604H	Eksma Optics	2	390,00	780,00
Rod holder base adapter	820-0225	Eksma Optics	1	6,00	6,00
Standard rod holder	820-0050	Eksma Optics	1	12,50	12,50
Hex key (10 pieces)	820-0270	Eksma Optics	1	7,30	7,30
Longpass filter (800 nm)	#66-059	Edmund Optics	2	28,50	57,00
Aespheric lenses		Thorlabs	2	60,00	120,00
Screw (10 pieces)	820-0560	Eksma Optics	1	7,30	7,30
Altera DE2 board (Academic discount)	-	Terasic	1	253,78	253,78
Single photon counting module	SPCM-AQRH-11	Excelitas	2	3022,00	6044,00
Total (€)	11772,00				

In this case CrystaLaser offers an academic discount of 134€ that has been already taken into account when calculating the total price.

5.2.3 Option 3: CCU implemented in an electronic protoboard

The CCU is a crucial part of the quantum entanglement experiment. Several options are offered but we choose, in view of an experimental application, to build our own prototype. For assembling this CCU the necessary electronic materials are listed below.

Table 5.5. CCU elements

Optical element	Reference	Provider	Quantity	Price per unit (€)	Total (€)
Flip flop	74ACT74	Onda Radio	4	0,66	2,64
220 pF Capacitor	DS220P	Onda Radio	3	0,04	0,11
1kOhm resistor	CR011K	Onda Radio	3	0,03	0,08
51 ohm Resistor	CR0151H	Onda Radio	2	0,03	0,05
Protoboard	-	Cetronic	1	4,95	4,95
Total (€)	7,84				

The CCUcircuit is, in principle, easy to build but will need some knowledge on electronics. Moreover, in order to extract results an oscilloscope will be also needed (see [50]).

Taking into account the pricelists provided before in **Table 5.3** and **Table 5.4** the total price will change as follows:

Table 5.6. Modified pricelists including a simpler CCU circuit

	Price (€)
Option 1. Thorlabs laser	10891
Option 2. CrystaLaser laser	11519

In both cases there is a saving of 253 € corresponding to the Altera DE2 board. As already mentioned in previous chapters some of the optical elements can be replaced by daily objects. For example, apertures – diaphragms – can be replaced by an aluminum sheet and, the beam stop, by a piece of black velvet cloth. This means that apertures could be removed from the pricelists leaving as a result what is next:

Table 5.7. Modified pricelists including a simpler CCU circuit and removing diaphragms and beam trap.

	Price (€)
Option 1. Thorlabs laser	10644
Option 2. CrystaLaser laser	11272

This time there is a saving of 500 € corresponding to Altera DE2 board, diaphragms, beam trap, post holders, clamping traps and adapters.

Since both prices are similar and the main difference stands on the laser it should be taken into account that the laser provided by Crystalaser is already mounted so there is no need of manipulations for the purchaser. Since the laser is one of the most important elements for carrying out the experiments this option would be the most suitable since it ensures a well mounted laser module. Both providers are reliable and no bad reviews of them have been found. Nevertheless, Thorlabs takes 2-3 days for shipping the purchased elements while Crystalaser needs 3-4 weeks.

Finally, **Table 5.8** contains all the different prices for the different options available depending on the laser providers, if the CCU is simpler or more sophisticated and also the price in case of opting for using daily objects for replacing some of the elements.

Table 5.8. Summary table of the different options available

Option	Thorlabs laser	Crystalaser laser	Altera DE2	CCU circuit	Daily objects	Final price (€)
1	x		x			11144
2		x	x			11772
3	x			x		10891
4		x		x		11519
5	x				x	10644
6		x			x	11272

As already discussed before, the options including Crystalaser would be the most suitable in case of a narrow experience in the manipulation of optical elements. Since the usage of the CCU circuit implies having an oscilloscope and the university already has these devices this option should be considered since it supposes a saving of almost 300€ corresponding to Altera DE2 board. And, finally, as daily objects can also be used for replacing some of the optical elements that would mark a difference on the price of 500€ when compared to the option number 2 which is the most expensive. In case of a tight budget Option 6 would be the best option to bet on.

CHAPTER 6. CONCLUSIONS

The main goal of this work was demonstrating the feasibility of assembling a quantum optical laboratory in the EETAC premises. In order to prove it was possible we started this work with an analysis of the field: the importance of quantum technology nowadays and for the future, the fact that it is a discipline that is currently expanding due to the infinite possibilities it offers when compared to classical methods and how it will become an outstanding way of performing communication channels that will change the current communication scenery we know.

With the objective of demonstrating the viability of this project it was necessary to introduce basic concepts for understanding the experiments we wanted to carry out as well as providing a proper explanation on the quantum basic properties that were put into practice later. Once the fundamental concepts described were set down, we define the functioning of the different elements that were going to be used in the experiments. So, an explanation on the way these elements work was also given. Quantum technologies have several intriguing properties such as entanglement and superposition that make them a powerful tool for bringing off different applications as teleportation. Along this work these applications have been analyzed and some experiments to put them into practice have also been provided.

Once the experiments to be performed were chosen, we found the necessary elements to make them possible. First of all, we start the source of light, the laser. Once the laser was fixed, the others were incorporated because the wavelength of the laser determines the filters, the crystals…and so on. It is worth saying that much information was consulted for ensuring a correct assembling: different papers, datasheets, books and other works on the field were necessary.

When the set-up elements were defined, we looked for providers that offered such elements. Different providers were checked in order to have a wide visibility on the different options in the market at our disposal. Needless to say that both affordable and expensive alternatives were proposed with the objective of better analyze what would suit better our needs.

At this stage we demonstrated that the feasibility of this quantum optical laboratory is real: we explained the basic theory, the functioning of the elements and even gave a list of providers for purchasing. Moreover, EETAC has appropriate spaces for undertaking this ambitious as well as challenging and interesting project taking also into account that some of the elements are already available in the laboratories. With a starting price of approximately 3000€ for the Quantum Cryptography kit the assembling of this quantum optical laboratory is feasible. It is an innovative initiative that will for sure encourage the new generation of undergraduate students develop their skills in this new and advantageous field that aims to become a leading discipline in the near future.

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ANNEX

1. Further work

1.1 Quantum Eraser

Another experiment we could perform using the kits supplied by Thorlabs is a Quantum Eraser whose principal actor will be the Mach-Zender. A Mach-Zender interferometer is a device mainly used for determining the different deviation of the phase that affects two light beams generated by the same source. It first came to light in 1891 and is attributed to the physicists Ludwig Mach and Ludwig Zehnder (see [48]). A basic diagram of a Mach-Zehnder is represented in **Figure A.1.**

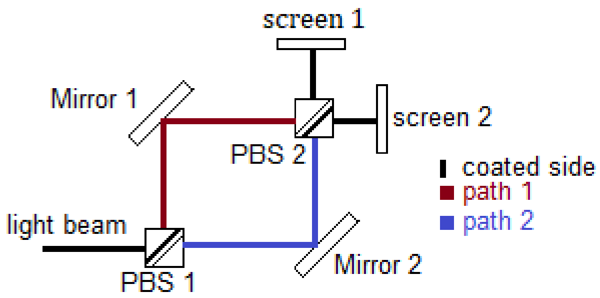

Figure A.1. Mach-Zender interferometer set-up.

In this experiment light will fall upon two screens – screen 1 and screen 2 – with the objective of seeing in a visual way the final phase to which the paths of light after crossing the two beam splitters will be subdued. The light that travels along the two paths and passes through the beam splitters and the mirror is subject to the addition of phases that, at the end, will result in a total phase that will be visible in the screens.

If we analyze first the contributions of the two paths in screen 2 separately **(Figure A.1)** we will get a total difference of 0 degrees. As path one is subjected to a total of two reflections and one transmission, the beam splitters act twice adding a phase of 2**π** **(A.1)** while the second path is transmitted once and reflected twice **(A.2)**. Note that the nomenclature $\Delta\Phi_{\text{path x\_y}}$ means the addition of phases to which the path x is subjected until it reaches screen y. Phase contributions over screen 2 will be first analyzed.

Reflection + reflection + transmission

$$\Delta\Phi_{\text{path 1\_2}} = \pi + \pi + 0 = 2\pi \qquad \textbf{(A.1)}$$

Transmission + reflection + reflection

$$\Delta\Phi_{\text{path 2\_2}} = 0 + \pi + \pi = 2\pi \tag{A.2}$$

Then, if we calculate the total contributions of phase in screen 1 it will be as follows:

$$\Delta\Phi_{\text{total\_screen2}} = \Delta\Phi_{\text{path 1\_2}} - \Delta\Phi_{\text{path 2\_2}} = 0 \tag{A.3}$$

Now, the light that reaches screen 1 will have a total phase of $\boldsymbol{\pi}$. The path number 1 will be reflected three times whereas **(A.4)** path 2 will be first transmitted then reflected and finally transmitted again **(A.5)**.

Reflection + reflection + reflection

$$\Delta\Phi_{\text{path 1\_1}} = \pi + \pi + 0 = 2\pi \tag{A.4}$$

Transmission + reflection + transmission

$$\Delta\Phi_{\text{path 2\_1}} = 0 + \pi + 0 = \pi \tag{A.5}$$

$$\Delta\Phi_{\text{total\_screen1}} = \Delta\Phi_{\text{path 1\_1}} - \Delta\Phi_{\text{path 2\_1}} = \pi \tag{A.6}$$

The last reflection in **(A.4)** attracts attention as it does not add a phase shift of π as one may have thought in a first stage. This absence of phase shift is due to the properties of the beam splitter. This crystal adds a phase of π only when light is reflected by the coated side of the crystal. In this case the incoming light is reflected by the uncoated side so no additional phase will be added (see [48]).

Finally, the difference between the two screens will be π **(A.6)** which is the biggest difference of phase that can be achieved when the paths of the Mach-Zehnder have the same length (see [48]).

$$\Delta\Phi_{\text{total\_screen1}} - \Delta\Phi_{\text{total\_screen2}} = \boldsymbol{\pi} \tag{A.7}$$

When a polarizer is placed in the trajectory of both paths and adjusted to the same polarization a spot of light with interferences – represented as a series of concentric rings – will reach both screens. In this case there is no way of knowing the path the photon at the input of the interferometer took and what we will clearly see in the screens is the difference of **π** between the two paths. On the other hand, when setting one of the polarizers so that there is a phase difference of 90 degrees respect the other the photons that reach the screen will be determined by the path they took. The result will be that the interfered spots of light that were observed before will become smoother, no concentric rings will be observed. Having this into account if a third polarizer – oriented 45 degrees with respect to the others – is placed between the last polarizing splitter and the screen the concentric rings will appear once again because the third polarizer that acts as an eraser sets the polarization of the photons to 45 degrees.

The provider puts at customer's disposal a useful complete guide that gathers quite interesting information about the experiments to perform, safety measures and the way to assemble the experiments.

1.2 Quantum Eraser Kit

The Quantum Eraser kit is also available in two versions depending on the metrics. In our case the metric version will be picked. Analogously **Table A.1** summarizes all the elements that will be included when the kit is purchased.

Table A.1 Quantum Cryptography kit elements, quantities and prices

Optical element	Quantity	Price per unit (€)	Total (€)
Laser module	1	144,00	144,00
Power supply	1	77,50	77,50
Mirror	2	12,50	25,00
Mirror mount	3	34,83	104,49
Adapter	1	20,40	20,40
Convex lens	1	20,79	20,79
Fixed lens mount	1	13,71	13,71
Beamsplitter	2	63,25	126,50
Beamsplitter mount	2	83,25	166,50
Polarizing film	1	7,45	7,45
Rotation mount	3	84,25	252,75
Screen	2	16,70	33,40
Mounting post (75 mm)	10	4,88	48,80
Mounting post (50 mm)	2	4,67	9,34
Post holder (75 mm)	9	7,44	66,96
Post holder (50 mm)	2	6,93	13,86
Pedestal post holder (80,90 mm)	1	22,30	22,30
Mounting base (25 mm x 58	8	5,04	40,32

mm x 10 mm)			
Mounting base (25 mm x 58 mm x 10 mm)	1	4,68	4,68
Mounting base (50 mm x 75 mm x 10 mm)	2	6,57	13,14
Clampling fork	1	8,15	8,15
Alignment tool	1	20,40	20,40
Rubber breadboad feet	4	4,73	18,92
Retaining ring	1	4,05	4,05
Adjustment Knobs (10 units)	4	23,20	92,80
Spanner Wrench	1	24,20	24,20
Breadboard (45 cm x 60 cm)	1	342,00	342,00
Hex key (3 mm)	1	NA	NA
Hex key (2 mm)	1	NA	NA
Hex key (1,3 mm)	1	NA	NA
Hex key (0,9 mm)	1	NA	NA
Ball driver	1	7,65	7,65
Washer	11	NA	NA
Nut	4	NA	NA
Screw (20 mm)	4	7,90	31,60
Screw (16 mm)	11	7,45	81,95
Screw (12 mm)	11	7,20	79,20
Total - Elements bought separately. NA not included	1922,81		
Kit total price	1661,00		

Table A.2 shows the elements not included in **Table A.1**. Then, the final price in case of buying the elements separately would be 2060 € which is almost 500 € beyond the price of the kit.

Table A.2 Quantum Cryptography elements to buy separately

Optical element	Reference	Quantity	Price per unit (€)	Total (€)
Hex key (3 mm)	HKM50NM	1	52,72	52,72
Hex key (2 mm)	HKM20NM	1	42,72	42,72
Hex key (1,3 mm)	HKM13NM	1	42,72	42,72
Total (€)	138,16			

In this case, as also happened with the Cryptography Kit, it is more beneficial to buy Thorlabs kit because it implies saving money, efforts and time.

2. Specifications Sheets

Table A.3 Specifications Sheets

Element	Provider	Link
405 nm laser diode	Thorlabs	https://www.thorlabs.com/thorproduct.cfm?partnumber=L405P20
405 nm laser	Crystalaser	http://www.crystallaser.com/new/bluelaser.html#405
SPCM	Excelitas	http://www.excelitas.com/downloads/DTS_SPCM-AQRH.pdf